UCM & GRAVITY: THE BIG PICTURE

Every problem is the same equation. The trick is knowing what to substitute.

Start here. Always.

$$\Sigma F = ma$$

Net Force = Mass × Acceleration

THE LEFT SIDE: What can ΣF become?

Forces add up. Some forces have special forms. Identify what's pushing or pulling.

Friction
$f_s \leq \mu_s F_N$

Tension
T or components
$T_x = T \sin\theta$

Gravity
$F_g = mg$ or
$F_g = \frac{GMm}{r^2}$

Normal
F_N

THE RIGHT SIDE: What can a become?

For circular motion in the **radial direction**:

$$a = \frac{v^2}{r}$$

Points toward the center. Always.

Key Insight: "F_c" is not a real force! It's shorthand for "net force toward center." On an FBD, never label a force F_c—identify the real forces (friction, tension, gravity, normal) that point inward.

PUTTING IT TOGETHER: The Master Equation

$$\Sigma F_{radial} = \frac{mv^2}{r}$$

Sum of forces toward center = mass × centripetal acceleration

Flat curve (car turning):
$f_s = \frac{mv^2}{r}$
Friction points to center

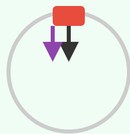
Conical pendulum:
$T \sin\theta = \frac{mv^2}{r}$
Horizontal component of tension

Satellite orbit:
$\frac{GMm}{r^2} = \frac{mv^2}{r}$
Gravity IS the centripetal force

Vertical loop (bottom):
$F_N - mg = \frac{mv^2}{r}$
Normal up, weight down

VERTICAL CIRCLES: Top vs. Bottom

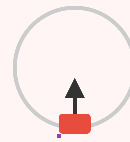
AT TOP: Same Direction



$$F_N + mg = \frac{mv^2}{r}$$

Both point toward center (down) → ADD

AT BOTTOM: Opposite Directions



$$F_N - mg = \frac{mv^2}{r}$$

F_N toward center, mg away → SUBTRACT

ORBITS: When Gravity Provides All the Centripetal Force

Set $F_g = F_c$:

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \rightarrow v = \sqrt{\frac{GM}{r}}$$

The satellite mass m cancels! Orbital speed depends only on M (central body) and r.

PROBLEM-SOLVING CHECKLIST

✓ Before you calculate:

1. **Draw the FBD.** What forces act on the object? Label them with real names (not F_c).
2. **Define "toward center."** Which direction is radial? That's your positive direction.
3. **Write $\Sigma F_{radial} = \frac{mv^2}{r}$.** Forces toward center are positive. Forces away are negative.
4. **Check your r.** Is it radius of circle, or something else? (Orbital r = center to center!)
5. **Solve.** Usually for v, F_N, T, or r.

PROPORTIONAL REASONING: The Substitution Method

Example: A satellite orbits at radius r with speed v. If it moves to radius $4r$, what's its new speed?

Step 1: Write the relationship: $v = \sqrt{\frac{GM}{r}}$

Step 2: Substitute new value: $v_{new} = \sqrt{\frac{GM}{4r}} = \sqrt{\frac{1}{4}} \cdot \sqrt{\frac{GM}{r}} = \frac{1}{2}v$

Answer: Speed becomes **half**. (Farther out = slower orbit)

The pattern: Substitute the change, keep constants as "GM" or "v", simplify to find the factor.

The Big Idea: There's no new physics here. It's $F = ma$ with specific substitutions for F and a. Master the substitutions, solve any problem.