Name: ______________ **AP Physics 1 — Day 5** Date: ______________

SATELLITE MOTION & ORBITS

WARM-UP

The ISS orbits 400 km above Earth. Astronauts inside appear to float weightlessly. A student claims: "*They float because there's no gravity in space.*"

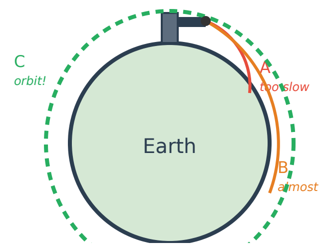
Do you agree or disagree? Explain your reasoning in 2-3 sentences.

PART 1: NEWTON'S CANNON

Isaac Newton proposed a thought experiment:

- Fire a cannonball horizontally; it curves down.
- Fire it faster; it goes further before hitting.
- Fire it **fast enough**, and the curve of the ball matches the curve of Earth.

Definition: An orbit is a projectile that is falling toward Earth, but moving so fast tangentially that it misses the ground.



PART 2: ORBITAL VELOCITY

To stay in orbit, the Force of Gravity must provide exactly the necessary Centripetal Force.

The Derivation (Memorize the logic, not the result!)

Set Universal Gravity equal to Centripetal Force:

$$F_g = F_c \quad \rightarrow \quad \frac{GMm}{r^2} = \frac{mv^2}{r}$$

M = Mass of Planet (Central Body) | m = Mass of Satellite | r = Orbital Radius (center to center)

ALGEBRA Solve for v

Rearrange the equation above to find the orbital speed v.

CONCEPT Mass Independence

Does the mass of the satellite m appear in the final answer?

If the ISS (400 tons) and a wrench (1 kg) are in the same orbit, which moves faster?

⚠ **Important:** This equation only works for **circular** orbits! Elliptical orbits require energy methods.

PART 3: PERIOD AND RADIUS

There's a precise relationship between how far a satellite orbits (r) and how long each orbit takes (T).

Step 1: Velocity from Geometry

Velocity is distance over time. For a circle, circumference $= 2\pi r$.

Write: Express v in terms of r and T.

Step 2: Substitute and Solve

Plug your expression for v into the orbital velocity equation: $v = \sqrt{GM/r}$.

Then: Square both sides and solve for T^2.

The Result: You should get:

$$T^2 = \frac{4\pi^2}{GM} r^3$$

Everything except r^3 is constant for a given central body. **Key insight:** Farther out = longer period (and by a LOT—it's cubed!).

PART 4: WEIGHING THE COSMOS

How do we know the mass of the Earth? We can't put it on a scale. But we can "weigh" it by watching how its gravity affects orbiting objects.

Constants You'll Need:

$G = 6.67 \times 10^{-11}\ \frac{\mathrm{N{\cdot}m^2}}{\mathrm{kg^2}}$ $M_E = 6.0 \times 10^{24}$ kg $R_E = 6.4 \times 10^{6}$ m

Strategy: If we know the Period T and Radius r of a satellite, we can find the Mass M of the object it orbits.

$$T^2 = \frac{4\pi^2 r^3}{GM} \quad \rightarrow \quad M = \frac{4\pi^2 r^3}{GT^2}$$

Weighing the Earth

Find the mass of Earth by observing its satellite: **The Moon**. Data: $r = 3.84 \times 10^{8}$ m, $T = 27.3$ days.

Step 1: Convert T to seconds.

→

Step 2: Solve for M_{Earth}.

Pause and appreciate this. Once we knew G (measured in a laboratory in 1798), every astronomical observation became a scale. We never visited the Moon's orbit with a ruler. We just watched how long it took to go around, did some algebra, and learned Earth's mass.

HOMEWORK

1. Weighing the Sun

Find the mass of the Sun by observing its satellite: **The Earth**. Data: $r = 1.50 \times 10^{11}$ m, $T = 365.25$ days.

a. Convert T to seconds.

b. Solve for M_{Sun}.

c. How many Earths equal the mass of the Sun? (Divide M_{Sun} by $M_{Earth} = 6.0 \times 10^{24}$ kg)

HOMEWORK (CONTINUED)

2. Geostationary Orbits

A "Geostationary" satellite stays above the same spot on Earth's equator. Its period must match Earth's rotation: $T = 24$ hours.

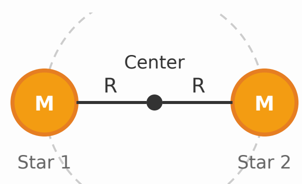
a. Convert T to seconds:

b. Solve for orbital radius r:

c. Calculate Altitude: Your radius is measured from Earth's *center*. What is the satellite's height above the surface? ($R_E = 6.4 \times 10^6$ m)

3. Binary Stars (Challenge)

Two stars of equal mass M orbit their common center of mass, located halfway between them.



a. Write F_g between the two stars. (Hint: What's the distance between them?)

b. Set $F_g = F_c$ and solve for v. (Hint: What's the radius of each star's orbit?)