

SHM DAY 5: LINEARIZATION

Warm-Up: You measure the period of a mass-spring system for 5 different masses and plot T vs m . The graph is a *curve* (a square root). Can you tell if your data matches $T = 2\pi\sqrt{m/k}$ just by looking at a curve? What if you could turn it into a straight line?

The Big Idea: A straight line is the most powerful shape in physics. If data falls on a line, you can: **(1)** confirm your model, **(2)** extract physical quantities from the slope, **(3)** spot bad data points. **The trick:** rewrite your equation in $y = mx + b$ form.

EXAMPLE: MASS-SPRING SYSTEM

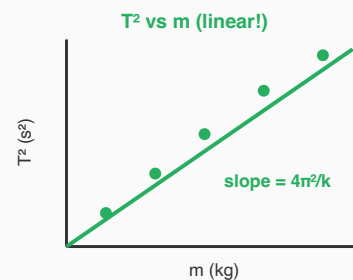
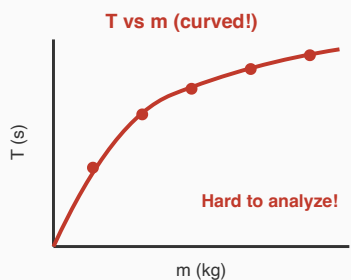
WE DO Linearizing $T = 2\pi\sqrt{m/k}$

Step 1: Square both sides.

$$T^2 = 4\pi^2 \cdot \frac{m}{k} \implies \underbrace{T^2}_y = \underbrace{\frac{4\pi^2}{k}}_{\text{slope}} \cdot \underbrace{m}_x$$

Step 2: Identify the axes. Plot T^2 (vertical) vs m (horizontal). This should give a straight line through the origin.

Step 3: Read the slope. slope = $\frac{4\pi^2}{k}$, so $k = \frac{4\pi^2}{\text{slope}}$.



YOU DO Practice: Extract k from a Slope

A student plots T^2 vs m for a spring and gets a best-fit line with slope = $2.5 \text{ s}^2/\text{kg}$. Find k .

LINEARIZING THE PENDULUM

WE DO Linearizing $T = 2\pi\sqrt{L/g}$

- (a) Square both sides. Write the equation in $y = mx$ form, identifying what goes on each axis.
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- (b) What is the slope in terms of physical constants?
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- (c) How would you extract g from the slope?
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THE GENERAL LINEARIZATION RECIPE

3 Steps: (1) Start with your equation. (2) Rearrange to $y = m \cdot x$ form (measured quantity = constants $\times$ controlled quantity). (3) Identify axes and what the slope means physically.

YOU DO Linearization Practice

For each equation, identify what to plot on each axis to get a straight line, and what the slope represents.

Equation	Plot $y =$	vs $x =$	Slope =
$T = 2\pi\sqrt{m/k}$	T^2	m	$4\pi^2/k$
$T = 2\pi\sqrt{L/g}$			
$v = \sqrt{2gh}$			
$F = kx$			
$T^2 = \frac{4\pi^2}{GM}r^3$			

Common Mistake: Plotting T vs $\sqrt{m}$ instead of T^2 vs m . Both work, but squaring the dependent variable is standard and less error-prone.

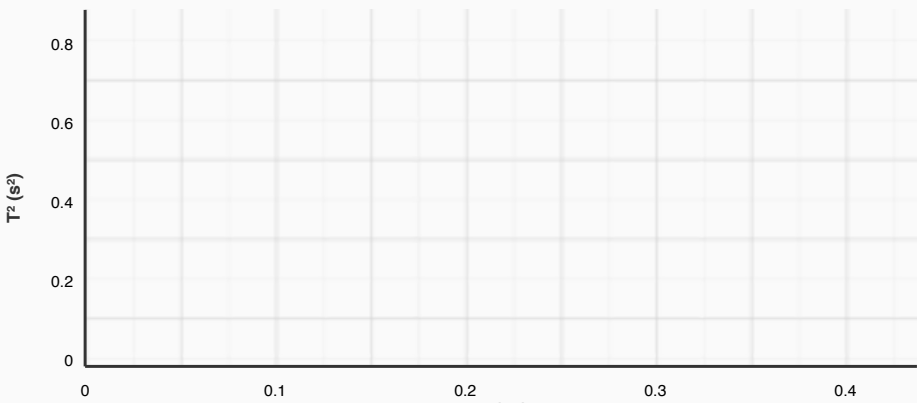
WORKING WITH REAL DATA

WE DO Spring Data Analysis

A student measures the period of a mass-spring system for different masses:

m (kg)	0.100	0.200	0.300	0.400	0.500
T (s)	0.44	0.63	0.77	0.89	0.99
T^2 (s ²)					

- (a) Complete the T^2 row by squaring each period.
- (b) On the grid below, plot T^2 (vertical) vs m (horizontal). Draw a best-fit line.



- (c) Calculate the slope using two points on your best-fit line (not data points):
- (d) Extract k from the slope:

THINK Quality Check

- (e) Your line should pass through (or very near) the origin. If it doesn't, what systematic error might be present? (Hint: does the spring itself have mass?)

PENDULUM DATA PRACTICE

YOU DO

Pendulum Data Analysis

A student measures the period of a pendulum at different lengths:

L (m)	0.25	0.50	0.75	1.00	1.25
T (s)	1.00	1.42	1.73	2.01	2.24
T^2 (s ²)					

(a) Complete the T^2 row. (b) Calculate the expected slope $4\pi^2/g$. (c) Estimate the slope from data. Does it match?

Exit Ticket

A student collects orbital data for Jupiter's moons: orbital radius r and period T . They want to find Jupiter's mass using $T^2 = \frac{4\pi^2}{GM} r^3$.

(a) What should they plot on each axis? (b) What does the slope represent? How do they extract M ?

HOMEWORK: PREP FOR LAB

1. In tomorrow's lab, you'll measure T for different masses on a spring. How many oscillations should you time? What should you keep constant?
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2. You get a slope of $1.85 \text{ s}^2/\text{kg}$ from your T^2 vs m plot. Calculate k . If the accepted value is $k = 22.0 \text{ N/m}$, find your percent error.
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3. A student's T^2 vs L graph has a y -intercept of $+0.15 \text{ s}^2$ instead of 0. Propose a physical explanation.
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Next Class: 2-day lab starts tomorrow. Day 1 = springs, Day 2 = pendulums. Bring a pencil and ruler for graphing.