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ANGULAR MOMENTUM IN ORBIT

Warm-Up (2 min): From your UCM unit, you already know that gravity provides centripetal force and $v = \sqrt{GM/r}$. New question: does an orbiting satellite have angular momentum? If so, can anything change it?

QUICK RECALL FROM UCM

$$F_g = \frac{GMm}{r^2}, \quad v = \sqrt{\frac{GM}{r}}, \quad T = \frac{2\pi r}{v}, \quad L = mvr$$

Paradox: Higher orbit = larger r = SLOWER speed... but MORE angular momentum (because r grows faster than v shrinks: $L = m\sqrt{GMr}$).

WHY ANGULAR MOMENTUM IS CONSERVED IN ORBITS

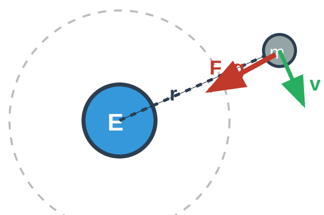
ZERO TORQUE FROM GRAVITY

Gravity pulls the satellite directly toward Earth's center. The position vector $\vec{r}$ points from Earth to the satellite. So $\vec{F}$ is antiparallel to $\vec{r}$:

$$\tau = rF\sin(180^\circ) = 0$$

Zero torque → L is conserved.

This is why orbits are stable. No torque means no change in angular momentum — the satellite keeps orbiting at the same L forever (unless thrusters fire).



F along r → sin(180°) = 0 → τ = 0

WE DO Comparing Orbital Angular Momentum

Data: ISS orbit r = 6.78 × 10^6 m, mass = 420,000 kg. Geostationary orbit r = 4.22 × 10^7 m, satellite mass = 3000 kg. Use $v = \sqrt{GM/r}$, then $L = mvr$.

(a) ISS: v = _____ m/s, L = _____ kg·m²/s

(b) Geostat: v = _____ m/s, L = _____ kg·m²/s

(c) The geostationary satellite is slower but in a higher orbit. Per unit mass, which orbit has more L/m? Why?

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WHAT CHANGES EARTH'S SPIN?

WE DO **Earth's Spin Angular Momentum**

Model Earth as a uniform solid sphere: $I = \frac{2}{5}MR^2$. Earth mass = 5.97 × 10^{24} kg, radius = 6.37 × 10^6 m, rotation period = 24.0 hours.

(a) Find ω in rad/s:

(b) Find I_{Earth}:

(c) Find $L_{spin} = I\omega$:

Physics in the Wild: Earthquakes Shorten Your Day

The 2011 Japan earthquake (M 9.1) shifted so much mass toward Earth's axis that it **decreased Earth's moment of inertia**. No external torque acted on Earth, so $L = I\omega$ is conserved. If I drops, ω must increase. Result: **the day shortened by 1.8 microseconds** and Earth's axis shifted by 17 cm. The 2004 Indian Ocean earthquake did the same thing — 2.68 microseconds faster.

YOU DO **Earthquake Calculation**

Suppose an earthquake decreases Earth's moment of inertia by a fraction δ (so $I_{new} = I(1-\delta)$). Angular momentum is conserved.

(a) Show that $\omega_{new} = \frac{\omega}{1-\delta}$.

(b) The 2011 quake shortened the day by 1.8 μs out of 86,400 s. What is the fractional change $\delta = \Delta I / I$? (Hint: $\Delta\omega/\omega \approx \delta$ for small δ.)

Physics in the Wild: The Three Gorges Dam

China's Three Gorges Dam holds 39.3 billion kg of water at ~175 m elevation. Raising that mass farther from Earth's spin axis **increased I**, so ω decreased. NASA calculated it **slowed Earth's rotation by 0.06 microseconds per day**. Same physics as a figure skater extending their arms — except the "arms" are billions of tons of water.

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THE MOON IS LEAVING US — 3.8 CM EVERY YEAR

1. Earth rotates once every 24 hours. The Moon orbits once every 27.3 days. Earth spins much faster than the Moon orbits.

2. The Moon's gravity raises tidal bulges on Earth — one facing the Moon, one on the far side.

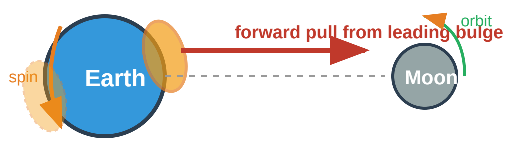
3. Because Earth spins faster, friction drags the bulges slightly **AHEAD** of the Earth-Moon line.

4. The leading bulge is closer to the Moon. Its gravitational pull has a small **forward component** — in the direction of the Moon's orbital motion.

5. This forward pull does positive work on the Moon, adding energy → larger orbit. **The Moon spirals outward at 3.8 cm/year.**

6. Angular momentum is conserved (no external torques on the Earth-Moon system). Moon gains orbital L → Earth loses spin L. **Earth's rotation slows by ~2.3 ms/century. Our days are getting longer.**

7. How we know: Apollo astronauts left retroreflector mirrors on the Moon. We bounce lasers off them and measure the distance to millimeter precision — 3.8 cm/year increase confirmed. Ancient Babylonian eclipse records from 2700 years ago show eclipses at locations that only work if the day was shorter then.



Bulge leads → forward pull → Moon gains ene

ANALYSIS The Conservation Connection

$L_{total} = I_{Earth}\omega_{Earth} + M_{Moon}v_{Moon}r_{orbit}$ = constant.

(a) If the Moon's orbital L increases, what must happen to Earth's spin L?

(b) Earth's I doesn't change (same mass, same shape). So what happens to ω_{Earth}?

(c) The Moon always shows us the same face. This means its spin period equals its orbital period — it's "tidally locked." Explain why this happened to the Moon first (think about which body is easier to slow down).

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EXIT TICKET & HOMEWORK

Exit Ticket: Asteroid Impact

A 5.0×10^{12} kg asteroid strikes Earth tangentially (in the direction of Earth's rotation) at 20 km/s. Model it as delivering angular impulse $\Delta L = mvR_{Earth}$ to Earth.

(a) Calculate ΔL delivered to Earth. ($R_{Earth} = 6.37 \times 10^6$ m)

(b) Earth's spin $L = 7.1 \times 10^{33}$ kg·m²/s. By what fraction does L change?

(c) Would the day get longer or shorter? By roughly how many seconds?

HOMEWORK

1. Moon vs. Earth: Moon mass = 7.35×10^{22} kg, orbital radius = 3.84×10^8 m, orbital speed = 1022 m/s.

(a) Calculate $L_{Moon} = mvr$.

(b) Compare to Earth's spin $L = 7.1 \times 10^{33}$ kg·m²/s. Which is larger? By what factor?

2. The 3.8 cm Question: The Moon is currently 3.84×10^8 m away and receding at 3.8 cm/year.

(a) By what percentage does the orbital radius increase per year?

(b) How many years until the Moon is 10% farther than it is now?

3. Conceptual — Satellite Boost: A satellite fires thrusters to move to a higher orbit. Its speed DECREASES but its angular momentum INCREASES. Explain how both can be true simultaneously. (Hint: