ORBITAL ENERGY AND ESCAPE

Zero at infinity, circular-orbit energy, and escape

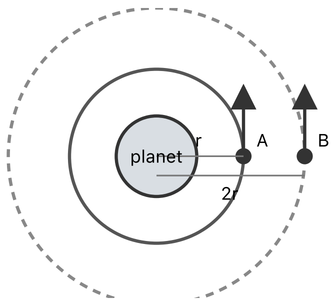
PREDICT

Two identical satellites move in circular orbits around the same planet. Satellite A is at radius r; satellite B is at radius $2r$. Which statement do you expect to be true?

A. $E_B < E_A$ **B.** $E_B = E_A$ **C.** $E_B > E_A$ **D. Both energies are zero**

Commit to a choice and be ready to defend it using the meaning of a bound system.

SAME SATELLITE, TWO POSSIBLE CIRCULAR ORBITS



Choose the zero before interpreting the sign

For orbital problems, define $U_g = 0$ when the two masses are infinitely far apart. At every finite separation,

$$U_g = -\frac{GMm}{r}$$

A negative total mechanical energy means the system is bound. Zero total energy is the limiting escape case.

On the diagram, circle the orbit with gravitational potential energy closer to zero. Explain your choice without calculating.

CIRCULAR ORBIT: ONE FORCE CONDITION, THREE ENERGIES

For a circular orbit, gravity is the radial net force:

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \quad \Rightarrow \quad v^2 = \frac{GM}{r}$$

Substitute this relation into $K = \frac{1}{2}mv^2$:

$$K_{circ} = \frac{GMm}{2r} = -\frac{U_g}{2}$$

$$E_{circ} = K + U_g = -\frac{GMm}{2r} = \frac{U_g}{2}$$

Quantity	Sign	What happens as circular-orbit radius increases?
U_g		
K		
E_{total}		
v	positive magnitude	

WE DO A 900 kg satellite moves in a circular orbit of radius 8.0×10^6 m around a planet of mass 6.0×10^{24} kg. Find U_g, K, and E_{total}. Use $G = 6.67 \times 10^{-11}$ N·m²/kg².

Explain why a thruster must add energy to move the satellite to a higher circular orbit even though the final orbital speed is smaller.

ESCAPE IS THE ZERO-ENERGY BOUNDARY

MINIMUM ESCAPE ENERGY ACCOUNT

For the minimum escape speed, the object arrives infinitely far away with no kinetic energy remaining:

$$\frac{1}{2}mv_{esc}^2 - \frac{GMm}{R} = 0$$

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$

The escaping object's mass cancels. At an altitude *h*, use the center-to-center radius $r = R + h$.

WE DO A planet has $M = 3.0 \times 10^{24}$ kg and $R = 4.0 \times 10^{6}$ m. Find the escape speed from its surface. Ignore atmosphere and rotation.

Initial total energy	Long-term classification
$E < 0$	
$E = 0$	
$E > 0$	

A probe is launched at $0.80v_{esc}$. Without calculating a decimal speed, determine the sign of its total energy and whether it escapes.

MODEL CHECK: RADIUS, ENERGY, AND ESCAPE

1. A satellite moves from a circular orbit of radius r to one of radius $3r$. Find E_f/E_i. State whether the system's total energy increases or decreases.

2. A planet has $M = 5.0 \times 10^{24}$ kg and $R = 7.0 \times 10^{6}$ m. Find the surface escape speed. Carry units through the substitution.

3. A probe of mass m is launched from radius R at $0.80v_{esc}$. Express its total energy as a multiple of GMm/R, then classify the motion.

4. The same satellite is initially in a circular orbit at r_i and later in a circular orbit at $r_f > r_i$. Complete the comparisons and justify each one from an equation.

Quantity	Higher orbit compared with lower orbit	Equation or reasoning
U_g		
K		
E_{total}		
v		

Exit: In one sentence, distinguish the zero of gravitational potential energy from the zero-energy escape threshold.