ANSWER KEY — ROTATION DAY 9: ROTATIONAL KINETIC ENERGY & WORK

Energy in Rotation | For teacher use only

WARM-UP

Flywheel Energy Storage

(a) Where is the energy stored? **In the rotating motion (rotational kinetic energy).**

The flywheel has no translational motion (it stays in one place), no change in height, and isn't a spring. But it's spinning at high speed, so it stores energy as $K_{\text{rot}} = \frac{1}{2}I\omega^2$.

(b) If you doubled the spin rate: **The energy would MORE than double.**

Since $K \propto \omega^2$, doubling ω gives a factor of 4 increase in energy. This is why flywheels are so effective for energy storage — small speed increases yield huge energy gains.

FLYWHEEL ENERGY (PAGE 1)

Solid steel cylinder: M = 50 kg, R = 0.40 m, ω = 3000 rpm

(a) Find I = ½MR²

$$I = \frac{1}{2}(50)(0.40)^2 = \frac{1}{2}(50)(0.16) = 4.0 \text{ kg·m}^2$$

I = 4.0 kg·m²

(b) Convert 3000 rpm to rad/s

$$\omega = 3000 \text{ rpm} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} \times \frac{1 \text{ min}}{60 \text{ s}} = \frac{3000 \times 2\pi}{60} = 100\pi = 314.2 \text{ rad/s}$$

ω = 314 rad/s (or 100π rad/s)

(c) Find K_rot

$$K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}(4.0)(314.2)^2 = 2.0 \times 98,722 = 197,444 \text{ J}$$

K_rot ≈ 197 kJ or 1.97 × 10^5 J

(d) If ω doubles, by what factor does K_rot change?

Since $K \propto \omega^2$, doubling ω gives $(2)^2 = 4$ times more energy.

$$K_{\text{rot}}(\text{doubled}) = \frac{1}{2}I(2\omega)^2 = 4 \times \frac{1}{2}I\omega^2 = 4K_{\text{rot}}(\text{original})$$

Factor of 4

SPINNING UP A DISK (PAGE 2)

Constant torque: τ = 8.0 N·m, I = 0.60 kg·m², rotates 5.0 rev from rest

(a) Calculate W = τΔθ

First, convert revolutions to radians:

$$\Delta\theta = 5.0\text{ rev} \times \frac{2\pi}{1\text{ rev}} = 10\pi = 31.4\text{ rad}$$

$$W = \tau\Delta\theta = 8.0 \times 31.4 = 251\text{ J}$$

W = 251 J (or 80π J)

(b) Use work-energy to find ω_f

$$W_{\text{net}} = \Delta K_{\text{rot}} = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2$$

$$251 = \frac{1}{2}(0.60)\omega_f^2 - 0$$

$$\omega_f^2 = \frac{251 \times 2}{0.60} = \frac{502}{0.60} = 836.7$$

$$\omega_f = \sqrt{836.7} = 28.9\text{ rad/s}$$

ω_f ≈ 29 rad/s

BRAKING A WHEEL (PAGE 2)

Wheel: I = 1.2 kg·m², ω_i = 30 rad/s, comes to rest

(a) How much work does the brake do?

$$W = \Delta K = K_f - K_i = 0 - \frac{1}{2}(1.2)(30)^2$$

$$W = -\frac{1}{2}(1.2)(900) = -540\text{ J}$$

W = −540 J (negative because friction removes energy)

(b) Brake torque 6.0 N·m; through how many radians?

$$|W| = |\tau|\Delta\theta$$

$$540 = 6.0 \times \Delta\theta$$

$$\Delta\theta = 90\text{ rad}$$

Δθ = 90 rad

GRAPHICAL: WORK FROM T VS Θ GRAPH (PAGE 2)

Wheel with I = 0.80 kg·m², starting from rest

Graph: τ = 12 N·m from θ = 0 to 3π rad; τ = 6 N·m from 3π to 6π rad

(a) Work from θ = 0 to θ = 3π

$$W_1 = \tau_1 \Delta\theta_1 = 12 \times 3\pi = 36\pi = 113 \text{ J}$$

W_1 = 113 J

(b) Work from θ = 3π to θ = 6π

$$W_2 = \tau_2 \Delta\theta_2 = 6 \times 3\pi = 18\pi = 56.5 \text{ J}$$

W_2 = 56.5 J

(c) Total work and ω at θ = 6π

$$W_{\text{total}} = 113 + 56.5 = 169.5 \text{ J}$$

Using work-energy theorem:

$$W_{\text{total}} = \frac{1}{2} I\omega^2$$

$$169.5 = \frac{1}{2}(0.80)\omega^2$$

$$\omega^2 = \frac{169.5 \times 2}{0.80} = \frac{339}{0.80} = 423.75$$

$$\omega = 20.6 \text{ rad/s}$$

W_total = 169.5 J | ω = 20.6 rad/s

ENERGY BAR CHARTS (PAGE 3)

Disk Dropping on Axle

Disk: m = 2.0 kg, R = 0.10 m, drops h = 1.5 m

(a) Set up energy conservation and find v

Initial: $U_g = mgh$, $K_{\text{trans}} = 0$, $K_{\text{rot}} = 0$
Final: $U_g = 0$, $K_{\text{trans}} = \frac{1}{2}mv^2$, $K_{\text{rot}} = \frac{1}{2}I\omega^2$
Conservation: $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$
With $I = \frac{1}{2}MR^2$ and $v = \omega R$:

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v}{R}\right)^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{4}mv^2 = \frac{3}{4}mv^2$$

(b) Solve for v:

$$v = \sqrt{\frac{4gh}{3}} = \sqrt{\frac{4(9.80)(1.5)}{3}} = \sqrt{\frac{58.8}{3}} = \sqrt{19.6} = 4.43 \text{ m/s}$$

v = 4.43 m/s

Ball Rolling Down Ramp (Page 3)

Solid sphere: m = 3.0 kg, R = 0.08 m, h = 2.0 m

(a) Write energy conservation equation

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

With $I = \frac{2}{5}MR^2$ and $v = \omega R$:

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v}{R}\right)^2 = \frac{1}{2}mv^2 + \frac{1}{5}mv^2 = \frac{7}{10}mv^2$$

(b) Find bottom speed

$$v = \sqrt{\frac{10gh}{7}} = \sqrt{\frac{10(9.80)(2.0)}{7}} = \sqrt{\frac{196}{7}} = \sqrt{28} = 5.29 \text{ m/s}$$

v = 5.29 m/s

(c) Compare with sliding block

A block sliding (no friction): $v = \sqrt{2gh} = \sqrt{2(9.80)(2.0)} = \sqrt{39.2} = 6.26 \text{ m/s}$

The block is faster because it doesn't "waste" energy on spinning. All the gravitational PE becomes translational KE. The sphere must share its energy between translation and rotation.

EXIT TICKET (PAGE 4)

Both objects: m = 1.0 kg, R = 0.20 m, ω = 10 rad/s

(a) Find K_rot for each

Hoop: $I_{\text{hoop}} = MR^2 = 1.0(0.20)^2 = 0.04 \text{ kg·m}^2$

$$K_{\text{hoop}} = \frac{1}{2}(0.04)(10)^2 = 0.02 \times 100 = 2.0 \text{ J}$$

Disk: $I_{\text{disk}} = \frac{1}{2}MR^2 = \frac{1}{2}(1.0)(0.20)^2 = 0.02 \text{ kg·m}^2$

$$K_{\text{disk}} = \frac{1}{2}(0.02)(10)^2 = 0.01 \times 100 = 1.0 \text{ J}$$

K_hoop = 2.0 J | K_disk = 1.0 J

(b) Which stores more energy, and why?

The hoop stores more energy (2.0 J vs 1.0 J).

The hoop has larger rotational inertia ($I = MR^2$ vs $I = \frac{1}{2}MR^2$) because all its mass is at the rim, far from the axis. This means it takes more energy to spin it at the same angular velocity, so it stores more.

HOMEWORK

1. Basic Rotational KE

Thin rod rotating about end: m = 0.80 kg, L = 1.2 m, ω = 5.0 rad/s

$$I = \frac{1}{3}ML^2 = \frac{1}{3}(0.80)(1.2)^2 = \frac{1}{3}(0.80)(1.44) = 0.384 \text{ kg·m}^2$$

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}(0.384)(5.0)^2 = 0.192 \times 25 = 4.8 \text{ J}$$

K_rot = 4.8 J

2. Rotational Work

Torque: τ = 15 N·m, I = 0.50 kg·m², from rest to 120 rad/s
(a) Find K_rot at 120 rad/s:

$$K = \frac{1}{2}(0.50)(120)^2 = 0.25 \times 14,400 = 3,600 \text{ J}$$

(b) Find angle turned (using W = ΔK):

$$W = 15 \times \Delta\theta = 3,600$$

$$\Delta\theta = 240 \text{ rad}$$

(a) K = 3600 J | (b) Δθ = 240 rad

3. Rolling Energy Split

Solid cylinder rolling at v = 4.0 m/s (so ω = v/R)
For a solid cylinder, $I = \frac{1}{2}MR^2$

$$K_{\text{total}} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v}{R}\right)^2$$

$$K_{\text{total}} = \frac{1}{2}mv^2 + \frac{1}{4}mv^2 = \frac{3}{4}mv^2$$

(a) Fraction that is translational:

$$\frac{K_{\text{trans}}}{K_{\text{total}}} = \frac{\frac{1}{2}mv^2}{\frac{3}{4}mv^2} = \frac{2}{3} = 66.7\%$$

(b) Fraction that is rotational:

$$\frac{K_{\text{rot}}}{K_{\text{total}}} = \frac{\frac{1}{4}mv^2}{\frac{3}{4}mv^2} = \frac{1}{3} = 33.3\%$$

(a) 66.7% or 2/3 translational | (b) 33.3% or 1/3 rotational

4. Looking Ahead: Will a bowling ball or basketball reach the bottom first?

Both reach at the same time (assuming they have the same radius).

The formula is $v = \sqrt{\frac{2gh}{1+c}}$ where $c = I/(MR^2)$. For a solid sphere (which both a bowling ball and basketball approximate), $c = 2/5$, regardless of mass. So both have the same shape constant and thus the same speed at the bottom. The bowling ball is heavier, but that doesn't change the speed — inertia (both translational and rotational) scales with mass in the same way.