ANSWER KEY — ROTATION DAY 8: NEWTON'S 2ND LAW FOR ROTATION

τ = Iα | For teacher use only

WARM-UP

Grinding Wheel Stopping

$\omega_0 = 40\ \text{rad/s}$, $\omega_f = 0$, $t = 8.0\ \text{s}$

(a) Angular acceleration:

$$\alpha = \frac{\omega_f - \omega_0}{t} = \frac{0 - 40}{8.0} = -5.0\ \text{rad/s}^2$$

$\alpha = -5.0\ \text{rad/s}^2$

(b) Net torque:

$$\Sigma\tau = I\alpha = 0.50 \times (-5.0) = -2.5\ \text{N·m}$$

$\tau = -2.5\ \text{N·m}$

(c) Friction force (R = 0.20 m):

$$\tau = FR \implies F = \frac{|\tau|}{R} = \frac{2.5}{0.20} = 12.5\ \text{N}$$

F = 12.5 N

EXERCISE 1: HAND PULLING A STRING ON A FLYWHEEL

(a) Find I for the disk

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(6.0)(0.20)^2 = \frac{1}{2}(6.0)(0.04) = 0.12\ \text{kg·m}^2$$

$I = 0.12\ \text{kg·m}^2$

(b) Torque from the string pull

The string pulls tangentially at the rim with force F = 15 N.

$$\tau = FR = 15 \times 0.20 = 3.0\ \text{N·m}$$

$\tau = 3.0\ \text{N·m}$

(c) Apply Στ = Iα to find α

$$\alpha = \frac{\Sigma\tau}{I} = \frac{3.0}{0.12} = 25\ \text{rad/s}^2$$

$\alpha = 25\ \text{rad/s}^2$

(d) Find ω after 3.0 s starting from rest

$$\omega = \omega_0 + \alpha t = 0 + 25(3.0) = 75 \text{ rad/s}$$

ω = 75 rad/s

(e) Number of revolutions in 3.0 s

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = 0 + \frac{1}{2}(25)(3.0)^2 = \frac{1}{2}(25)(9.0) = 112.5 \text{ rad}$$

$$\text{Revolutions} = \frac{112.5}{2\pi} = \frac{112.5}{6.283} = 17.9 \text{ rev}$$

17.9 revolutions

What If: Pulling at half the radius

(a) The torque: $\tau = FR = 15(0.10) = 1.5 \text{ N·m}$ — **Torque decreases by half**

(b) The angular acceleration: $\alpha = 1.5/0.12 = 12.5 \text{ rad/s}^2$ — **Angular acceleration also decreases by half**

EXERCISE 2: MASSIVE PULLEY (ATWOOD MACHINE)

System: Pulley (M = 4.0 kg, R = 0.15 m), m_1 = 3.0 kg (left), m_2 = 5.0 kg (right)

(a) Write ΣF = ma for m_1 (assume UP is +)

$$T_1 - m_1 g = m_1 a$$

$$T_1 - 3.0(9.80) = 3.0a$$

$T_1 = 29.4 + 3.0a$

(b) Write ΣF = ma for m_2 (assume DOWN is +)

$$m_2 g - T_2 = m_2 a$$

$$5.0(9.80) - T_2 = 5.0a$$

$T_2 = 49.0 - 5.0a$

(c) Write Στ = Iα for the pulley

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(4.0)(0.15)^2 = 0.045 \text{ kg·m}^2$$

The net torque is $\tau_{\text{net}} = T_2 R - T_1 R = (T_2 - T_1)R$ (CCW is positive).

$$(T_2 - T_1)R = I\alpha$$

$(T_2 - T_1)(0.15) = 0.045\alpha$

(d) Solve for acceleration using the constraint a = αR

From the pulley equation: $\alpha = \frac{(T_2 - T_1)(0.15)}{0.045} = 3.33(T_2 - T_1)$

Constraint: $a = \alpha R = \alpha(0.15) \implies \alpha = a/0.15$

So: $\frac{a}{0.15} = 3.33(T_2 - T_1)$
Substitute the tensions:

$$\frac{a}{0.15} = 3.33\left[(49.0 - 5.0a) - (29.4 + 3.0a)\right]$$
$$\frac{a}{0.15} = 3.33(19.6 - 8.0a)$$
$$\frac{a}{0.15} = 65.27 - 26.67a$$
$$a = 9.79 - 4.0a$$
$$5.0a = 9.79$$
$$a = 1.96\ \mathrm{m/s^2}$$

a = 1.96 m/s² or 2.0 m/s²

EXERCISE 2 (CONTINUED): THE EFFECTIVE MASS SHORTCUT

(a) Effective mass for I = ½MR²

$$M_{\mathrm{eff}} = \frac{I}{R^2} = \frac{\frac{1}{2}MR^2}{R^2} = \frac{1}{2}M$$

$M_{\mathrm{eff}} = \frac{1}{2}M = 0.5M$

(b) Numerical value of M_eff for the 4.0 kg disk

$$M_{\mathrm{eff}} = \frac{1}{2}(4.0) = 2.0\ \mathrm{kg}$$

M_eff = 2.0 kg

(c) Use the system equation to find a

$$a = \frac{m_2 g - m_1 g}{m_1 + m_2 + M_{\mathrm{eff}}} = \frac{(5.0 - 3.0)(9.80)}{3.0 + 5.0 + 2.0}$$
$$a = \frac{2.0 \times 9.80}{10.0} = \frac{19.6}{10.0} = 1.96\ \mathrm{m/s^2}$$

a = 1.96 m/s² or 2.0 m/s² — matches Page 3 result!

What If: Hollow Hoop (I = MR²)?

(a) Effective mass: $M_{\mathrm{eff}} = I/R^2 = MR^2/R^2 = M = 4.0\ \mathrm{kg}$
(b) Acceleration would be less. With a larger M_{eff}, the denominator increases:

$$a' = \frac{19.6}{3.0 + 5.0 + 4.0} = \frac{19.6}{12.0} = 1.63\ \mathrm{m/s^2}$$

The hoop pulley produces less acceleration because its rotational inertia (being distributed farther out) acts like more linear mass in the system.

EXERCISE 3: WINCH LIFTING A BUCKET

Winch: M = 8.0 kg, R = 0.12 m, τ_applied = 25 N·m; Bucket: m = 15 kg

(a) For the bucket (up is +):

$$T - mg = ma$$

$$T - 15(9.80) = 15a$$

$$T = 147 + 15a$$

(b) For the winch:

$$I = \frac{1}{2}(8.0)(0.12)^2 = 0.0576 \text{ kg·m}^2$$

$$\tau_{\text{app}} - TR = I\alpha$$

$$25 - T(0.12) = 0.0576\alpha$$

(c) Solve using a = αR:
$M_{\text{eff}} = I/R^2 = 0.0576/0.0144 = 4.0 \text{ kg}$

$$a = \frac{\tau_{\text{app}}/R - mg}{m + M_{\text{eff}}} = \frac{25/0.12 - 15(9.80)}{15 + 4.0}$$

$$a = \frac{208.3 - 147}{19} = \frac{61.3}{19} = 3.23 \text{ m/s}^2$$

a ≈ 3.2 m/s² (The bucket accelerates upward)

EXIT TICKET: BASIC T = IA

Solid disk: m = 2.0 kg, R = 0.10 m, T = 5.0 N

(a) Find I:

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(2.0)(0.10)^2 = 0.01 \text{ kg·m}^2$$

(b) Find τ:

$$\tau = TR = 5.0(0.10) = 0.50 \text{ N·m}$$

(c) Find α:

$$\alpha = \frac{\tau}{I} = \frac{0.50}{0.01} = 50 \text{ rad/s}^2$$

I = 0.01 kg·m² | τ = 0.50 N·m | α = 50 rad/s²

HOMEWORK

1. Simple Application

$$\alpha = \frac{\tau}{I} = \frac{12}{0.80} = 15 \text{ rad/s}^2$$

$$\omega = \alpha t = 15(3.0) = 45 \text{ rad/s}$$

$$\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}(15)(9.0) = 67.5 \text{ rad} = \frac{67.5}{2\pi} = 10.7 \text{ rev}$$

(a) α = 15 rad/s² | (b) ω = 45 rad/s | (c) 10.7 revolutions

2. Friction Brake

(a) Braking torque: $\tau = FR = 40(0.30) = 12 \text{ N·m}$

(b) Angular acceleration: $\alpha = -\tau/I = -12/2.5 = -4.8 \text{ rad/s}^2$

(c) Time to stop: $t = \frac{\omega_f - \omega_i}{\alpha} = \frac{0-120}{-4.8} = 25 \text{ s}$

(a) τ = 12 N·m | (b) α = −4.8 rad/s² | (c) t = 25 s

3. Coupled System

Mass m = 1.0 kg hanging from solid disk: M = 3.0 kg, R = 0.08 m

$$I = \frac{1}{2}(3.0)(0.08)^2 = 0.0096 \text{ kg·m}^2$$

$$M_{\text{eff}} = \frac{I}{R^2} = \frac{0.0096}{0.0064} = 1.5 \text{ kg}$$

$$a = \frac{mg}{m + M_{\text{eff}}} = \frac{1.0(9.80)}{1.0 + 1.5} = \frac{9.80}{2.5} = 3.92 \text{ m/s}^2$$

$$T = m(g - a) = 1.0(9.80 - 3.92) = 5.88 \text{ N}$$

Note: $T \neq mg$ because the disk's rotational inertia holds it back!

(a) a = 3.9 m/s² | (b) T = 5.9 N