NEWTON'S 2ND LAW FOR ROTATION

Warm-up (3 min): A grinding wheel (I = 0.50 kg·m²) is spinning at 40 rad/s. A brake pad presses against the rim and exerts a friction force. The wheel stops in 8.0 s.

(a) What is α?

(b) What is the net torque on the wheel?

(c) If the wheel radius is 0.20 m, what friction force does the brake pad exert?

THE ROTATIONAL NEWTON'S SECOND LAW

THE COMPLETE PARALLEL

Linear Motion: $\Sigma F = ma$ → Net force causes linear acceleration

Rotational Motion: $\Sigma\tau = I\alpha$ → Net torque causes angular acceleration

$$\Sigma\tau = I\alpha$$

Where: $\Sigma\tau$ = net torque (N·m), I = rotational inertia (kg·m²), α = angular acceleration (rad/s²)

Key Insight: This is Newton's Second Law in disguise. Force → Torque. Mass → Rotational inertia. Acceleration → Angular acceleration. Same physics, different geometry.

Physics in the Wild: Why EVs Win Off the Line

An electric motor delivers maximum torque at zero RPM — instantly. A gas engine needs to rev up first. So when the light turns green, the EV applies enormous τ to wheels with the same I, producing much larger α right from the start. That's why a 5000-lb Tesla can beat a Porsche in a drag race: Στ = Iα, and the EV has more τ when it matters most.

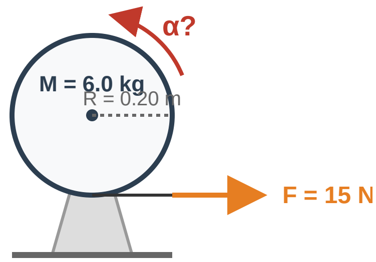
PROBLEM-SOLVING: ROTATION WITH Τ = IΑ

1. Identify the rotating object and its axis
2. Find I (from table or calculate using I = ½MR² for a disk)
3. Identify all torques (with signs: CCW = +, CW = −)
4. Write $\Sigma\tau = I\alpha$
5. Solve for the unknown

EXERCISE 1: HAND PULLING A STRING ON A FLYWHEEL

WE DO **Pure Rotation — No Hanging Mass**

A solid disk (mass 6.0 kg, radius 0.20 m) is mounted on a frictionless horizontal axle. A string is wrapped around the rim and pulled with a constant force of 15 N.



(a) Find I for the disk. ($I = \frac{1}{2}MR^2$)

I = _____ kg·m²

(b) The string pulls tangentially at the rim. Find the torque: $\tau = FR =$ _____

(c) Apply $\Sigma\tau = I\alpha$ to find α.

α = _____ rad/s²

(d) Starting from rest, what is ω after 3.0 s?

(e) How many revolutions does the disk complete in those 3.0 s?

Notice: This was a one-object problem. The string force creates a torque, the disk has rotational inertia, and $\tau = I\alpha$ gives us the angular acceleration. No hanging mass, no coupled equations.

QUICK **What If?**

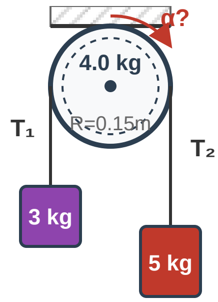
If the same 15 N force were applied at half the radius (wrapping the string around a smaller inner spool), what would happen to:

(a) The torque? (increase/decrease/same) **(b)** The angular acceleration? (increase/decrease/

EXERCISE 2: MASSIVE PULLEY (THE FORMAL METHOD)

WE DO **Coupled Rotation & Translation**

A pulley of mass 4.0 kg and radius 0.15 m (solid disk, $I = \frac{1}{2}MR^2$) is mounted on a frictionless axle. A light string passes over the pulley and connects a 3.0 kg mass (left) to a 5.0 kg mass (right).



Key Insight: $T_1 \neq T_2$ when the pulley has mass! It requires a net torque $(T_2R - T_1R)$ to accelerate the pulley's rotational inertia. You must write **three separate equations**.

(a) Write $\Sigma F = ma$ for m_1 (3.0 kg). Assume it accelerates UP (+).

(b) Write $\Sigma F = ma$ for m_2 (5.0 kg). Assume it accelerates DOWN (+).

(c) Write $\Sigma \tau = I\alpha$ for the pulley. (Hint: $\tau_{net} = T_2R - T_1R$ and $I = \frac{1}{2}MR^2$).

(d) The string ties the system together, so $a = \alpha R$ (or $\alpha = a/R$). Substitute α, combine your three equations to cancel out T_1 and T_2, and solve for the acceleration a.

a = ______ m/s²

Three coupled equations? A whole page of algebra? Yikes.
Is there a better way? Turn the page...

EXERCISE 2 (CONTINUED): THE "EFFECTIVE MASS" SHORTCUT

EFFECTIVE LINEAR MASS

The useful idea is more general than this pulley problem. Any time a **linear force** pulls on a rotating object at radius R, start with:

$$\Sigma\tau = I\alpha \quad \longrightarrow \quad FR = I\left(\frac{a}{R}\right)$$

Now divide both sides by R:

$$F = \left(\frac{I}{R^2}\right)a$$

So whenever rotation is linked to linear motion through $a = \alpha R$, the rotational inertia shows up in the linear equation exactly like an added mass of $\frac{I}{R^2}$. That is the **Effective Linear Mass (M_{eff})**.

SYSTEM SHORTCUT **Re-solving the Atwood Machine**

Because the pulley's rotational inertia acts like an extra mass holding the string back, we can unroll the entire Atwood machine and treat it as a single linear system:

$$a = \frac{\Sigma F_{external}}{m_{system}} = \frac{m_{heavy}g - m_{light}g}{m_{heavy} + m_{light} + M_{eff}}$$

(a) What is the formula for the effective mass (I/R^2) of our solid disk pulley ($I = \frac{1}{2}MR^2$)?

(b) Calculate the numerical value of M_{eff} for our 4.0 kg disk.

M_{eff} = _____ kg

(c) Plug everything into the system equation above to find a. Does it match your answer from Page 3?

a = _____ m/s²

General rule: If a rotating object is being driven by a string, belt, or other tangential force and the motion is tied together by $a = \alpha R$, you can treat the rotational part as an extra linear mass of I/R^2. In this Atwood machine, that means the 4.0 kg solid disk behaves like an extra 2.0 kg in the denominator.

EXERCISE 3: WINCH LIFTING A BUCKET

YOU DO **Applied Torque + Translation**

A cylindrical winch (mass 8.0 kg, radius 0.12 m, $I = \frac{1}{2}MR^2$) is used to lift a 15 kg bucket. An applied torque of 25 N·m turns the crank.

(a) Write $\Sigma F = ma$ for the bucket (up = +): ________________________________

(b) Write $\Sigma\tau = I\alpha$ for the winch: ________________________________

(c) Use $a = \alpha R$ to solve for a:

EXIT **Basic τ = Iα**

A solid disk (mass 2.0 kg, radius 0.10 m) has a string wrapped around it. A constant tension of 5.0 N pulls on the string.

(a) Find I = _____ kg·m². **(b)** Find τ = _____ N·m. **(c)** Find α = _____ rad/s².

HOMEWORK

1 **Simple Application**

A torque of 12 N·m is applied to a wheel with I = 0.80 kg·m². (a) Find α. (b) Starting from rest, what is ω after 3.0 s? (c) How many revolutions in those 3.0 s?

2 **Friction Brake**

A flywheel (I = 2.5 kg·m², radius 0.30 m) spins at 120 rad/s. A brake applies a friction force of 40 N to the rim. (a) Find the braking torque. (b) Find α. (c) How long to stop?