Name: ____________ Date: ____________ Period: ____________

ROTATION DAY 7: ROTATIONAL INERTIA

Warm-Up (3 min): Prediction Task

A hoop and a solid disk have the same mass and radius. Both start from rest at the top of an identical ramp. Which reaches the bottom first?

(A) Hoop **(B) Disk** **(C) Same time** **(D) Not enough info**

Your prediction: ____________

Reasoning: ____

WHAT RESISTS ROTATION?

THE PARALLEL

In linear motion: Force causes acceleration. F = ma. More mass → harder to accelerate.

In rotation: Torque causes angular acceleration. $\tau = I\alpha$. But what is I?

Key insight: It's not just total mass — it's *where* the mass is located relative to the rotation axis.

ROTATIONAL INERTIA (MOMENT OF INERTIA)

$$I = \sum m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \cdots$$

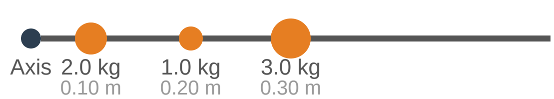
Where: m = mass of each piece (kg), r = distance from rotation axis (m)

Units: $kg \cdot m^2$

Key principle: Mass far from the axis contributes *much more* to I than the same mass near the axis (because of the r^2 dependence!).

WE DO Point Masses on a Rod

A lightweight rod has three masses attached: 2.0 kg at 0.10 m, 1.0 kg at 0.20 m, and 3.0 kg at 0.30 m from the rotation axis.



(a) Calculate I for the system.

I = ______ $kg \cdot m^2$

(b) If the 3.0 kg mass moves to 0.10 m, what is the new I?

I_{new} = ______ $kg \cdot m^2$

Name: ____________ Date: ____________ Period: ____________

COMMON SHAPES — THE REFERENCE TABLE

ROTATIONAL INERTIA OF STANDARD SHAPES

Shape	Axis	I
Point mass	Distance r from axis	mr^2
Thin hoop/ring	Through center	MR^2
Solid disk/cylinder	Through center	$\frac{1}{2}MR^2$
Solid sphere	Through center	$\frac{2}{5}MR^2$
Thin rod	Through center	$\frac{1}{12}ML^2$
Thin rod	Through end	$\frac{1}{3}ML^2$

Note: You don't need to memorize these — they'll be given on the AP exam. But understand the *pattern:* mass spread farther from the axis → larger I. That's why $I_{\text{hoop}} > I_{\text{disk}} > I_{\text{sphere}}$ (all same M and R).

Physics in the Wild: Biomechanics

Tightrope walker: Why carry a 10-meter pole? It puts mass *far* from the pivot (your feet), creating a huge I. Since $\alpha = \tau / I$, large I means you tip *slowly*, giving time to correct.

Sprinter: Why bend your knees when running? Bending brings mass *closer* to the hip joint, *decreasing* I, so your legs rotate faster for the same muscle torque.

SKILL Ranking Task

Rank these objects from **largest to smallest** rotational inertia. All have mass M and relevant dimension R or L.

(A) Solid disk about center **(B)** Hoop about center **(C)** Solid sphere about center **(D)** Point mass at distance R

Ranking: ____

WE DO Revisit the Warm-Up: Hoop vs. Disk Race

Both hoop and disk have mass M and radius R. Hoop: $I = MR^2$. Disk: $I = \frac{1}{2}MR^2$.

(a) Which has larger rotational inertia?

(b) Which resists angular acceleration more?

(c) On a ramp, gravitational PE converts to both translational KE ($\frac{1}{2}mv^2$) and rotational KE ($\frac{1}{2}I\omega^2$). The object with larger I "wastes" more energy on spinning. Which object has more translational speed at the

Name: ____________ Date: ____________ Period: ____________

QUALITATIVE REASONING: WHERE THE MASS SITS MATTERS

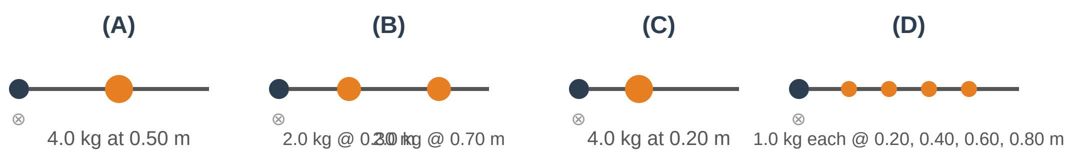
THE HAMMER GRIP TEST

Hold a hammer by the handle and waggle it back and forth. Now flip it and hold it by the head. **Which grip makes it easier to rotate?**

The total mass is the same either way — but the *distribution* of mass relative to your wrist (the axis) changes dramatically. When the heavy head is far away, I is large. When the heavy head is near the axis, I is small.

SKILL Ranking Task — Mass Distribution

All systems below have total mass $M = 4.0$ kg and rotate about the axis shown (⊗). Rank from **largest to smallest** rotational inertia. Justify each choice using the r^2 reasoning.



Ranking (largest I → smallest I): ______________________________

Justification:

WE DO Proportional Reasoning — How Does Doubling r Affect I?

A point mass m is at distance r from the axis: $I = mr^2$.

(a) If r doubles (mass stays the same), by what factor does I change?

(b) If m doubles (distance stays the same), by what factor does I change?

(c) A 2.0 kg mass sits at 0.30 m from the axis. You move it to 0.90 m. By what factor did I change? Calculate both values to verify.

Name: ____________________ **Date:** ____________________ **Period:** ____________________

EXIT TICKET & HOMEWORK

EXIT TICKET

Three objects roll down a ramp: a solid sphere, a solid cylinder (disk), and a hoop. All have the same mass and radius.

(a) Rank them by finish time (fastest to slowest):

(b) In one sentence, explain why the ranking doesn't depend on mass or radius.

HOMEWORK

1 Point Masses in a Square

Four 0.50 kg masses are at the corners of a square (side length 0.40 m). Find I about an axis through the center of the square, perpendicular to the square. (Hint: what is r for each mass?)

I = _____ kg·m²

2 Shape Comparison: Hoop vs. Disk

A hoop and a solid disk both have mass 3.0 kg and radius 0.20 m.

(a) Calculate I for each:

I_{hoop} = _____ kg·m², I_{disk} = _____ kg·m²

(b) A torque of 0.60 N·m is applied to each. Find α for each using τ = Iα (Day 8 preview!):

α_{hoop} = _____ rad/s², α_{disk} = _____ rad/s²

3 Axis Location Matters

A thin uniform rod (mass 2.0 kg, length 1.0 m) can rotate about (A) an axis through the center ($I = \frac{1}{12}ML^2$), or (B) an axis through one end ($I = \frac{1}{3}ML^2$).