ANSWER KEY — ROTATION DAY 6: TORQUE & STATICS QUIZ

Multiple Choice & Free Response | For teacher use only

PART I: MULTIPLE CHOICE (3 PTS EACH) — 15 POINTS TOTAL

1. Angular Kinematics

Given: $\alpha = 5.0\ \text{rad/s}^2$, $t = 4.0\ \text{s}$, initial rest so $\omega_0 = 0$

$\omega = \omega_0 + \alpha t = 0 + (5.0)(4.0) = 20\ \text{rad/s}$

Answer: **(B) 20 rad/s**

2. Linear-Angular Relationship

Analysis: Both P and Q rotate with the same ω. Linear speed is $v = \omega r$. Point P: $v_P = \omega R$. Point Q: $v_Q = \omega(R/2)$. So $v_P = 2v_Q$.

Answer: **(C) P has twice the linear speed of Q**

3. Torque Calculation

Given: $r = 0.40\ \text{m}$, $F = 60\ \text{N}$, force at 90° to wrench

$\tau = rF\sin(90^\circ) = (0.40)(60)(1) = 24\ \text{N·m}$

Answer: **(B) 24 N·m**

4. Equilibrium Conditions

Analysis: A couple is two equal forces applied at opposite ends in opposite directions. $\sum F = 0$ for a couple, but the torques do NOT cancel — there's a net torque. This disproves the claim that $\sum F = 0$ is sufficient for equilibrium.

Answer: **(C) Two equal forces applied at opposite ends of a beam in opposite directions (a couple)**

5. Rotational Kinematics

Given: $\omega_0 = 120\ \text{rad/s}$, $\omega_f = 0$, $t = 6.0\ \text{s}$

Find: $\Delta\theta$ using $\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

First find α: $\alpha = \frac{\omega_f - \omega_0}{t} = \frac{0-120}{6.0} = -20\ \text{rad/s}^2$

$\Delta\theta = (120)(6.0) + \frac{1}{2}(-20)(6.0)^2 = 720 - 360 = 360\ \text{rad}$

Answer: **(B) 360 rad**

PART II: FREE RESPONSE

FRQ 1: Torque Calculation (12 pts)

Setup: Meter stick (0.20 kg) pivoted at 30 cm mark. Hanging mass 0.50 kg at 10 cm mark.

(a) Free-body diagram (3 pts)

Forces on meter stick:

- Weight of hanging mass: $W_h = (0.50)(9.80) = 4.90\ \text{N}$ acting downward at 10 cm, distance from pivot = 20 cm (to the LEFT)
- Weight of meter stick: $W_s = (0.20)(9.80) = 1.96\ \text{N}$ acting downward at 50 cm mark, distance from pivot = 20 cm (to the RIGHT)
- Support force at pivot, pointing upward

Diagram shows: Pivot at 30 cm. Hanging mass 4.90 N at 20 cm left. Meter stick weight 1.96 N at 20 cm right. Support force upward at pivot.

(b) Net torque about pivot (4 pts)

$\tau_{\text{hanging}} = rF\sin(90^\circ) = (0.20)(4.90)(1) = 0.98\ \text{N·m}$ (clockwise, negative)
$\tau_{\text{stick}} = rF\sin(90^\circ) = (0.20)(1.96)(1) = 0.392\ \text{N·m}$ (counterclockwise, positive)
$\tau_{\text{net}} = 0.392 - 0.98 = -0.588\ \text{N·m}$ or $\approx -0.59\ \text{N·m}$
Net torque: $-0.59\ \text{N·m}$ (clockwise) — **NOT in equilibrium** (net torque $\neq 0$). The stick would rotate clockwise.

(c) Balancing mass placement (3 pts)

For equilibrium: $\tau_{\text{net}} = 0$. Place 0.30 kg mass at distance x to the RIGHT of pivot.
$\tau_{\text{weight}} + \tau_{\text{stick}} + \tau_{\text{added}} = 0$
$-0.98 + 0.392 + (0.30)(9.80)\cdot x = 0$
$2.94x = 0.588 \Rightarrow x = 0.20\ \text{m} = 20\ \text{cm}$
Position: $30\ \text{cm} + 20\ \text{cm} =$ **50 cm from the 0 end (or 20 cm to the right of the pivot)**

(d) Initial motion without the balancing mass (2 pts)

Since $\tau_{\text{net}} = -0.59\ \text{N·m}$ (clockwise, negative), the meter stick rotates **clockwise (the hanging mass side goes down)**.
The unbalanced torque from the hanging mass dominates.

FRQ 2: Shop Sign / Statics (15 pts)

Setup: Horizontal beam, 2.5 m long, 12 kg, hinged at left end. Cable at right end, 40° with beam. 25 kg sign at 2.0 m from wall.

(a) Weights (3 pts)

$W_{\text{beam}} = (12)(9.80) =$ **117.6 N**
$W_{\text{sign}} = (25)(9.80) =$ **245 N**

(b) Cable tension using torques about hinge (5 pts)

Taking torques about the hinge (counterclockwise positive):
$\sum \tau = 0$
$T\sin(40^\circ)\cdot(2.5) - W_{\text{beam}}\cdot(1.25) - W_{\text{sign}}\cdot(2.0) = 0$
$T(0.6428)(2.5) - (117.6)(1.25) - (245)(2.0) = 0$
$1.607T = 147 + 490 = 637$
$T =$ **396 N**

(c) Hinge force components (4 pts)

Horizontal: $H_x = T\cos(40^\circ) = (396)(0.766) =$ **303 N (to the right)**
Vertical: $\sum F_y = 0 \Rightarrow H_y + T\sin(40^\circ) = W_{\text{beam}} + W_{\text{sign}}$
$H_y = 117.6 + 245 - (396)(0.6428) = 362.6 - 254.6 =$ **108 N (upward)**

(d) Maximum sign mass (3 pts)

Max tension allowed = 500 N. From torque equation:
$500\sin(40^\circ)\cdot(2.5) = m_s g(2.0) + 117.6(1.25)$
$500(0.6428)(2.5) = m_s(9.80)(2.0) + 147$
$803.5 = 19.6m_s + 147$
$m_s = 33.5\ \text{kg}$ → **Maximum sign mass ≈ 33.5 kg** (or 34 kg)

FRQ 3: Angular Kinematics & Circular Motion (8 pts)

(a) Convert and find angular acceleration (3 pts)

$\omega_f = 8000\ \text{rpm} \times \frac{2\pi\ \text{rad}}{1\ \text{rev}} \times \frac{1\ \text{min}}{60\ \text{s}} = \frac{8000\times 2\pi}{60} =$ **837.76 rad/s** (or $\approx 840\ \text{rad/s}$)
$\alpha = \frac{\omega_f - \omega_0}{t} = \frac{837.76 - 0}{15} =$ **55.85 rad/s²** (or $\approx 56\ \text{rad/s}^2$)

(b) Revolutions during spin-up (3 pts)

$\Delta\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}(55.85)(15)^2 = \frac{1}{2}(55.85)(225) = 6283\ \text{rad}$
Number of revolutions: $\frac{6283}{2\pi} =$ **1000 revolutions** (or 1001)

(c) Centripetal acceleration (2 pts)

$a_c = \omega^2 r = (837.76)^2(0.12) = 702,854 \times 0.12 = 84,342\ \text{m/s}^2$

As multiple of g: $\frac{84{,}342}{9.80}$ $=$ **8,600g (or about 8600 times g)**