ROTATION DAY 5: TORQUE & STATICS REVIEW — KEY

ANSWER KEY — NOT FOR DISTRIBUTION

Convention: counterclockwise torque is positive. Using $g = 10$ m/s².

SKILL Quick Recall: Fill in the table

Quantity	Linear	Rotational
Position	x (m)	θ **(rad)**
Velocity	v (m/s)	ω **(rad/s)**
Acceleration	a (m/s²)	α **(rad/s²)**
Inertia	m (kg)	I **(kg·m²) — Day 7**
Net cause of change	$\sum F = ma$	$\sum \tau = I\alpha$ **— Day 8**

1 Kinematics Review

$\omega_0 = 20$ rad/s, $\alpha = -4.0$ rad/s².

(a) How long until it stops?

$\omega = \omega_0 + \alpha t \rightarrow 0 = 20 + (-4.0)\,t \rightarrow t = 20/4.0$

$t = 5.0$ **s**

(b) How many revolutions?

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = 20(5.0) + \frac{1}{2}(-4.0)(5.0)^2 = 100 - 50 = 50$ rad

$\text{rev} = 50/(2\pi) = 7.96$

≈ 8.0 **revolutions (50 rad)**

(c) Tangential speed of rim point ($r = 0.30$ m)?

$v = \omega r = 20 \times 0.30$

$v = 6.0$ **m/s**

2 Torque Calculation

$F = 40$ N, $r = 0.50$ m, angle 55° from wrench handle.

(a) Calculate the torque.

$\tau = rF\sin\theta = (0.50)(40)\sin 55^\circ = 20 \times 0.819$

$\tau \approx 16.4$ **N·m**

(b) At what angle would the torque be maximum?

$\theta = 90^\circ$ **(force perpendicular to wrench)**

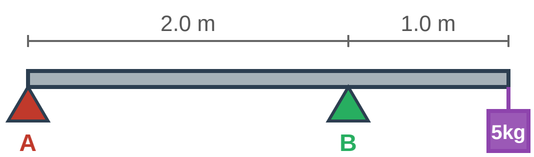
(c) What would the torque be at that angle?

$$\tau_{\max} = rF\sin 90^\circ = (0.50)(40)(1)$$

$\tau_{\max} = 20$ **N·m**

3 Statics — Supported Beam

3.0 m beam (10 kg → 100 N at center, x = 1.5 m). Support A at x = 0 (left end). Support B at x = 2.0 m. Hanging mass 5.0 kg → 50 N at x = 3.0 m (right end).



(a) Choose pivot at **A** — eliminates the unknown N_A (zero moment arm).

Pivot at A (left end). This eliminates N_A from the torque equation.

(b) $\sum \tau_A = 0$:

$$N_B(2.0) - 100(1.5) - 50(3.0) = 0$$

$$2.0\,N_B = 150 + 150 = 300$$

$N_B = 150$ **N**

(c) $\sum F_y = 0$:

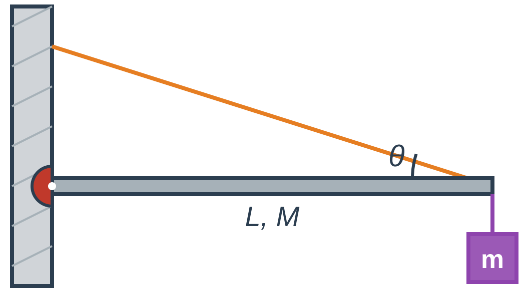
$$N_A + 150 - 100 - 50 = 0$$

$N_A = 0$ **N**

Why zero? The system's center of mass is at $x = (10 \times 1.5 + 5 \times 3.0)/(15) = 30/15 = 2.0$ m — directly over support B! Support A bears no load.

4 AP-Style FRQ Practice

Beam of length L, mass M, hinged to wall. Cable at angle θ from beam at free end. Mass m hangs from free end.



(a) Free-body diagram — five forces on the beam:

1. Mg downward at $L/2$ (beam weight at center)
2. mg downward at L (hanging mass)
3. Tension T along cable at L (components: $T\cos\theta$ horizontal, $T\sin\theta$ vertical)
4. H_x horizontal at hinge
5. H_y vertical at hinge

(b) Derive T. Take torques about hinge (eliminates H_x and H_y):

$$\sum \tau_{\text{hinge}} = 0: \quad T\sin\theta \cdot L - Mg \cdot \frac{L}{2} - mg \cdot L = 0$$

$$T\sin\theta = \frac{Mg}{2} + mg = \frac{g(M+2m)}{2}$$

$$T = \frac{g(M+2m)}{2\sin\theta}$$

As $\theta \to 0, T \to \infty$. A nearly horizontal cable must have enormous tension!

(c) Cable is cut — what happens?

The beam swings downward (clockwise) about the hinge. The initial angular acceleration is found from $\sum \tau = I\alpha$:

$$Mg \cdot \frac{L}{2} + mg \cdot L = \left(\frac{1}{3}ML^2 + mL^2\right)\alpha$$

The beam's moment of inertia about the hinge end is $\frac{1}{3}ML^2$, and the point mass contributes mL^2. This is a Day 8 topic — the setup is the key skill here.

5 Final Practice — Tipping vs. Sliding

$m = 50$ kg, $H = 2.0$ m, $W = 0.80$ m, $\mu_s = 0.30$. Horizontal push at top edge.

Sliding threshold:

$$F_{\text{slide}} = \mu_s \cdot mg = 0.30 \times 500 = 150 \text{ N}$$

Tipping threshold: Pivot at front bottom corner.

$$\sum \tau = 0: \quad F \cdot H = mg \cdot \frac{W}{2}$$

$$F(2.0) = 500(0.40) \quad \Rightarrow \quad F_{\text{tip}} = 200/2.0 = 100 \text{ N}$$

Compare:

$F_{\text{tip}} = 100$ N < $F_{\text{slide}} = 150$ N

The block tips first at $F = 100$ N (before it could slide at 150 N).

General rule: Compare W/H to $2\mu_s$. If $W/H < 2\mu_s$ → tips first. Here: $0.80/2.0 = 0.40 < 0.60 = 2(0.30)$. Tall, narrow objects tip; short, wide objects slide.