Rotation Day 4: Static Equilibrium - Step-by-Step Key

Use this as a teacher key / student check guide. Convention used: counterclockwise torque is positive.

Warm-Up

(a) Uniform beam, weight 200 N, length 3.0 m on one support.

1. For a uniform beam, the center of mass is at the midpoint.
2. To balance on one support, place support directly under the center of mass.

Support location: 1.5 m from either end (the middle).

(b) Add 100 N weight at right end, support remains in middle.

1. The new 100 N force at the right end creates a clockwise torque about the middle support.
2. Net torque is no longer zero, so the beam rotates clockwise (right side down).

Yes, it tips clockwise (right side drops).

(c) Restore balance without moving support.

1. Add a counterclockwise torque to cancel the new clockwise torque.
2. For example: add weight on the left side at appropriate distance.

Example: add a left-side counterweight (or remove/reposition right-side load).

Page 2 - Beam on Two Supports (We Do)

4.0 m beam, beam weight = 300 N at center (2.0 m), extra load = 500 N at 1.0 m from left support A. Supports at A (left) and B (right).

1. Take torques about A to eliminate unknown N_A.

$$\Sigma\tau_A = 0: \quad N_B(4.0) - 500(1.0) - 300(2.0) = 0$$

$$4N_B - 500 - 600 = 0 \;\rightarrow\; 4N_B = 1100 \;\rightarrow\; N_B = 275 \text{ N}$$

2. Use vertical force balance.

$$\Sigma F_y = 0: \quad N_A + N_B - 500 - 300 = 0$$

$$N_A + 275 - 800 = 0 \;\rightarrow\; N_A = 525 \text{ N}$$

N_B = 275 N, N_A = 525 N.

Check: 525 + 275 = 800 N = total downward force.

Page 2 - Off-Center Load (You Do)

Same beam, but 500 N load is now at 3.0 m from left end.

1. Torque about A:

$$\Sigma\tau_A = 0: \quad N_B(4.0) - 500(3.0) - 300(2.0) = 0$$

$$4N_B - 1500 - 600 = 0 \;\rightarrow\; 4N_B = 2100 \;\rightarrow\; N_B = 525 \text{ N}$$

2. Vertical force balance:

$$\Sigma F_y = 0: \quad N_A + N_B - 800 = 0$$

$$N_A + 525 - 800 = 0 \;\rightarrow\; N_A = 275 \text{ N}$$

N_B = 525 N, N_A = 275 N.

Comparison: moving the 500 N load toward B increases N_B and decreases N_A.

Page 3 - Shop Sign with Cable (We Do)

Beam length 2.0 m, beam weight 80 N at center, sign 150 N at far end, cable at far end making 30° above horizontal, hinge at wall.

1. Take torque about hinge so hinge forces drop out.
2. Only cable's vertical component contributes torque: $T_y = T \sin 30°$.

$$\Sigma\tau_{hinge} = 0: \quad (T \sin 30°)(2.0) - 150(2.0) - 80(1.0) = 0$$

$$2(T \sin 30°) - 300 - 80 = 0 \;\rightarrow\; T \sin 30° = 190$$

$$T = 190 / (\sin 30°) = 380 \text{ N}$$

3. Horizontal force balance:

$$\Sigma F_x = 0: \quad H_x - T \cos 30° = 0 \;\rightarrow\; H_x = 380 \cos 30° = 329 \text{ N}$$

4. Vertical force balance:

$$\Sigma F_y = 0: \quad H_y + T \sin 30° - 150 - 80 = 0$$

$$H_y + 190 - 230 = 0 \;\rightarrow\; H_y = 40 \text{ N}$$

T = 380 N, H_x = 329 N (to the right), H_y = 40 N (up).

Page 3 - Ladder Against Wall (You Do)

Ladder length 5.0 m at 60° to ground, ladder weight 200 N at center, person 700 N at 4.0 m from bottom. Wall is frictionless.

1. Use bottom contact as pivot to eliminate N_g and friction *f*.
2. Find horizontal lever arms for weights:

$$x_{ladder} = (2.5) \cos 60° = 1.25 \text{ m}, \quad x_{person} = (4.0) \cos 60° = 2.0 \text{ m}$$

3. Wall normal N_w acts horizontally at top; moment arm is top height:

$$h = (5.0) \sin 60° = 4.33 \text{ m}$$

4. Torque equation (CCW positive):

$$\Sigma\tau_{bottom} = 0: \quad 200(1.25) + 700(2.0) - N_w(4.33) = 0$$

$$250 + 1400 = 4.33N_w \rightarrow N_w = 381 \text{ N}$$

5. Horizontal balance:

$$\Sigma F_x=0: \quad N_w - f = 0 \rightarrow f = 381 \text{ N}$$

6. Vertical balance:

$$\Sigma F_y=0: \quad N_g - 200 - 700 = 0 \rightarrow N_g = 900 \text{ N}$$

N_w = 381 N, f = 381 N, N_g = 900 N.

Page 4 - Exit Ticket

2.0 m uniform plank (100 N) extends 0.50 m past table edge. Find max distance a 400 N person can walk past edge before tipping.

1. At tipping point, pivot is the table edge.
2. Plank center is 1.0 m from end, so center is 0.50 m inside table from pivot.
3. Set clockwise = counterclockwise torques:

$$400(x) = 100(0.50)$$

$$x = 50 / 400 = 0.125 \text{ m}$$

Maximum overhang for person: 0.125 m = 12.5 cm past the edge.

Homework Key

1) Conceptual: Why can pivot be any point?

1. If an object is in rotational equilibrium, angular acceleration is zero.
2. That means net torque is zero regardless of pivot choice.

Any pivot works; equations are equivalent when $\Sigma\tau = 0$.

2) Beam: 6.0 m uniform beam, weight 400 N, supports at x = 1.0 m and x = 6.0 m, 300 N load at x = 0.

1. Let support forces be N_1 at x=1.0 and N_2 at x=6.0.
2. Torque about x=1.0 m:

$$\Sigma\tau_{x=1} = 0: \quad +300(1.0) + N_2(5.0) - 400(2.0) = 0$$

$$300 + 5N\_2 - 800 = 0 \;\rightarrow\; N\_2 = 100 \text{ N}$$

3. Vertical balance:

$$N\_1 + N\_2 - 700 = 0 \;\rightarrow\; N\_1 = 600 \text{ N}$$

N_1 = 600 N, N_2 = 100 N.

3) Sign: 3.0 m beam, weight 120 N at center, 200 N sign at end, cable at 45° at end.

1. Torque about hinge:

$$(T \sin 45°)(3.0) - 200(3.0) - 120(1.5) = 0$$

$$3T \sin 45° = 600 + 180 = 780$$

$$T \sin 45° = 260 \;\rightarrow\; T = 260 / (\sin 45°) = 368 \text{ N}$$

Cable tension: T ≈ 368 N.

4) AP preview counterexample: $\Sigma F = 0$ does not guarantee equilibrium.

1. Apply two equal and opposite forces at different points (a force couple).
2. Net force is zero, but net torque is nonzero, so object rotates.

Example: turning a steering wheel with two opposite tangential forces.