ROTATION DAY 3: TORQUE & LEVER ARM — KEY

ANSWER KEY — NOT FOR DISTRIBUTION

Convention: CCW = positive torque, CW = negative torque. Using $g = 10$ m/s².

WARM-UP Check the Rankings — Bolt Scenarios

Long wrench = 0.30 m. Short wrench = 0.15 m.

Strategy	r (m)	F (N)	θ	$\tau = rF\sin\theta$ (N·m)
A	0.30	50	90°	**15.0**
B	0.15	100	90°	**15.0**
C	0.30	100	30°	**15.0**
D	0.15	100	90°	**15.0**

A = B = C = D — all produce exactly 15.0 N·m!

Key insight: This is intentional! Torque depends on the *product* of all three factors (r, F, $\sin\theta$). You can compensate for a smaller value in one by increasing another. Students rarely predict all four are equal.

WILD Cheater Bar Quick Math

Mechanic applies 200 N with a 0.30 m wrench. Adds a 0.60 m pipe.

Original: $\tau = (0.30)(200)\sin 90^\circ = 60$ N·m

With pipe: $r = 0.30 + 0.60 = 0.90$ m → $\tau = (0.90)(200)(1) = 180$ N·m

Torque triples from 60 to 180 N·m. Tripling the lever arm triples the torque.

WE DO Beam with Two Forces

1.0 m beam pivoted at center. 20 N down at left end, 12 N down at right end.

(a) Torque from left force (20 N down, 0.50 m left of pivot):

Down on left → CCW (positive). $\tau_1 = +(20)(0.50) = +10$ N·m

(b) Torque from right force (12 N down, 0.50 m right of pivot):

Down on right → CW (negative). $\tau_2 = -(12)(0.50) = -6.0$ N·m

(c) Net torque:

$\sum \tau = +10 + (-6.0) = +4.0$ **N·m**

(d) Which way does the beam rotate?

CCW (positive net torque) — the left side goes down because it has more force.

YOU DO **Angled Force on a Door**

Door width 0.80 m, force applied at 0.60 m from hinge. $F = 45$ N at 60° to door surface.

(a) The lever arm is the perpendicular distance from the hinge to the line of action of the force.

(b) Torque about the hinge:

$$\tau = rF\sin\theta = (0.60)(45)\sin 60^\circ$$

$$\tau = 27 \times 0.866$$

$\tau \approx 23.4$ **N·m**

WE DO **Playground Seesaw**

Child A: 35 kg at 2.0 m left of pivot. Child B: 25 kg at d_B right of pivot.

(a) Torque from Child A's weight:

Weight: $W_A = 35 \times 10 = 350$ N. Down on left → CCW → positive.

$\tau_A = +(350)(2.0) = +700$ **N·m**

(b) Distance for Child B to balance ($\sum \tau = 0$):

Weight: $W_B = 25 \times 10 = 250$ N. Down on right → CW → negative.

$+700 - 250\, d_B = 0 \quad \Rightarrow \quad d_B = 700/250$

$d_B = 2.8$ **m**

Check: The lighter child must sit farther from the pivot. 35 kg × 2.0 m = 25 kg × 2.8 m = 70 kg·m ✓

YOU DO **Three Forces**

Beam pivoted at left end. 30 N down at 0.20 m, 50 N up at 0.60 m, 20 N down at 1.0 m.

(a) Calculate each torque (with sign):

30 N down at 0.20 m right of pivot: down on right → CW → negative.
$\tau_1 = -(30)(0.20) = -6.0$ N·m

50 N up at 0.60 m right of pivot: up on right → CCW → positive.
$\tau_2 = +(50)(0.60) = +30$ N·m

20 N down at 1.0 m right of pivot: down on right → CW → negative.
$\tau_3 = -(20)(1.0) = -20$ N·m

(b) Net torque:

$\sum \tau = -6.0 + 30 - 20 = +4.0$ **N·m**

(c) Which way does the beam tend to rotate?

CCW (positive net torque) — the right end swings upward.

EXIT TICKET & HOMEWORK

EXIT **Wrench Torque**

$r = 0.25$ m, $F = 80$ N.

(a) Perpendicular push:

$\tau = rF \sin 90^\circ = (0.25)(80)(1)$

$\tau = 20$ **N·m**

(b) Push at 45° to the wrench:

$\tau = (0.25)(80) \sin 45^\circ = 20 \times 0.707$

$\tau \approx 14.1$ **N·m**

HW 1 **Revolving Door**

Answer: (a) — pushing at the outer edge, perpendicular.

This maximizes both r (outer edge) and $\sin\theta$ ($\sin 90^\circ = 1$), giving the largest torque.

(b) loses $\sin\theta$, (c) loses r, (d) gives zero torque ($\sin 0^\circ = 0$).

HW 2 **Lever Arm Practice**

$r = 0.40$ m, $F = 120$ N, $\theta = 75^\circ$.

$\tau = rF \sin\theta = (0.40)(120) \sin 75^\circ = 48 \times 0.966$

$\tau \approx 46.4$ **N·m**

HW 3 **Two-Force Balance — Meter Stick**

Meter stick (0.15 kg) supported at 40 cm mark. 0.50 kg at 10 cm. Where to place 0.30 kg?

Pivot at 40 cm mark. Stick weight = $0.15 \times 10 = 1.5$ N at 50 cm (10 cm right of pivot).

0.50 kg mass = 5.0 N at 10 cm mark (30 cm left of pivot).

0.30 kg mass = 3.0 N at position x cm.

Take torques about the support (40 cm mark):

5.0 N, 30 cm left: down on left → CCW → $\tau = +(5.0)(0.30) = +1.5$ N·m

1.5 N, 10 cm right: down on right → CW → $\tau = -(1.5)(0.10) = -0.15$ N·m

For balance: the 0.30 kg mass must be to the right of the support (CW torque to offset the net CCW).

$+1.5 - 0.15 - 3.0 \cdot d = 0 \quad \Rightarrow \quad d = 1.35/3.0 = 0.45$ m right of pivot

Place the 0.30 kg mass at the 85 cm mark (40 + 45 = 85 cm).