ROTATION DAY 2: LINEAR–ANGULAR CONNECTIONS — KEY

ANSWER KEY — NOT FOR DISTRIBUTION

WARM-UP Two Ants on a Record

Ant A at 5 cm, Ant B at 15 cm. Record spins at a steady rate.

(a) Which ant completes a circle faster?

Neither — both complete a circle at the same rate (same period T, same ω).

(b) Which ant travels a greater distance per revolution?

Ant B — it's farther out, so it traces a larger circle ($s = 2\pi r$).

(c) Which ant has a greater speed?

Ant B — same ω, larger r, so $v = \omega r$ is larger.

(d) Both sweep the same ______ per second.

Angle (angular velocity ω).

WE DO The Record Player Revisited

Vinyl record, $r = 15$ cm = 0.15 m, 33⅓ rpm.

(a) Convert to rad/s:

$$33\tfrac{1}{3}\ \text{rpm} = \frac{100}{3} \times \frac{2\pi}{60} = \frac{10\pi}{9}$$

$\omega \approx 3.49$ **rad/s**

(b) Linear speed at the rim:

$$v = \omega r = 3.49 \times 0.15$$

$v \approx 0.524$ **m/s**

(c) Speed 5.0 cm from center:

$$v = \omega r = 3.49 \times 0.050$$

$v \approx 0.175$ **m/s**

(d) Verify ratio:

$v_{\text{rim}}/v_{5\text{cm}} = 0.524/0.175 = 3.0$. Radius ratio: $15/5 = 3.0$ ✓

The speed ratio equals the radius ratio — confirming $v = \omega r$.

YOU DO **Bicycle Wheel**

$r = 0.35$ m, $\omega = 10$ rad/s.

(a) Speed of a point on the rim:

$$v = \omega r = 10 \times 0.35$$

$v = 3.5$ **m/s**

(b) Tangential acceleration ($\alpha = 2.0$ rad/s²):

$$a_t = \alpha r = 2.0 \times 0.35$$

$a_t = 0.70$ **m/s²**

(c) Centripetal acceleration:

$$a_c = v^2/r = (3.5)^2/0.35 = 12.25/0.35$$

$a_c = 35$ **m/s²**

a_c is 50× larger than a_t! At high speeds, the centripetal acceleration dominates.

QUICK **Which Acceleration?**

Situation	$a_t = 0$?	$a_c = 0$?
Constant ω, circular path	Yes	No
Speeding up, circular path	No	No
Constant v, straight line	Yes	Yes

Key: a_t is zero when $\alpha = 0$ (constant speed). a_c is zero only on a straight line (infinite radius).

WE DO **Car Tire Speed**

$r = 0.32$ m, $v = 25$ m/s, brakes to stop in 5.0 s.

(a) ω of the tire:

$\omega = v/r = 25/0.32$

$\omega = 78.1$ **rad/s**

(b) Braking α:

$\alpha = \Delta\omega/\Delta t = (0 - 78.1)/5.0$

$\alpha = -15.6$ **rad/s²**

(c) Revolutions while braking:

$\theta = \frac{1}{2}(\omega_0 + \omega)t = \frac{1}{2}(78.1 + 0)(5.0) = 195$ rad

$\text{rev} = 195/(2\pi) = 31.1$

≈ 31 **revolutions**

YOU DO **Pulley System**

$r = 0.10$ m, mass drops at $a = 2.0$ m/s².

(a) Angular acceleration of the pulley:

$\alpha = a/r = 2.0/0.10$

$\alpha = 20$ **rad/s²**

(b) Mass drops 0.80 m from rest — radians turned:

$\theta = s/r = 0.80/0.10$

$\theta = 8.0$ **rad**

EXIT TICKET & HOMEWORK

EXIT **Meshing Gears**

Gear A: $r_A = 10$ cm = 0.10 m, 120 rpm. Gear B: $r_B = 30$ cm = 0.30 m.

(a) Linear speed at edge of Gear A:

$\omega_A = 120 \times 2\pi/60 = 4\pi \approx 12.6$ rad/s

$v = \omega_A r_A = 12.6 \times 0.10$

$v \approx 1.26$ **m/s**

(b) ω for Gear B (same edge speed):

$\omega_B = v/r_B = 1.26/0.30$

$\omega_B \approx 4.19$ **rad/s (= 40 rpm)**

(c) Which has more angular velocity? Which edge has more linear speed?

Gear A has more ω (smaller gear spins faster). Both edges have the same linear speed.

HW 1 **Merry-Go-Round**

$r = 3.0$ m, one revolution in 8.0 s.

(a) $\omega = 2\pi/T = 2\pi/8.0 = \pi/4 \approx 0.785$ rad/s

(b) $v_{\text{rim}} = \omega r = 0.785 \times 3.0 \approx 2.36$ m/s

(c) $v_{1.0} = \omega r = 0.785 \times 1.0 \approx 0.785$ m/s

$\omega \approx 0.785$ **rad/s,** $v_{\text{rim}} \approx 2.36$ **m/s,** $v_{1.0} \approx 0.79$ **m/s**

HW 2 **Centrifuge**

$r = 0.12$ m, 4000 rpm.

$\omega = 4000 \times 2\pi/60 = 400\pi/3 \approx 419$ rad/s

$a_c = \omega^2 r = (419)^2 \times 0.12 \approx 21{,}100$ m/s²

$a_c/g = 21{,}100/9.8 \approx 2{,}150$

$a_c \approx 21{,}100$ **m/s²** $\approx 2{,}150\, g$

HW 3 **Chain & Sprocket**

Pedal sprocket $r = 8.0$ cm, rear sprocket $r = 4.0$ cm, pedal at 60 rpm.

$\omega_{\text{pedal}} = 60 \times 2\pi/60 = 2\pi$ rad/s

Chain speed: $v = \omega_{\text{pedal}} \times r_{\text{pedal}} = 2\pi \times 0.08 = 0.16\pi$ m/s

$\omega_{\text{rear}} = v/r_{\text{rear}} = 0.16\pi/0.04 = 4\pi$ rad/s

$\omega_{\text{rear}} = 4\pi \approx 12.6$ **rad/s (= 120 rpm — doubles because rear sprocket is half the size)**