

GUIDED PROBLEM: GEAR RATIOS — KEY

ANSWER KEY — NOT FOR DISTRIBUTION

Gear A: $r_A = 1.0$ m, $\omega_A = 1.0$ rad/s, so $v_A = \omega_A r_A = 1.0$ m/s (always).

GUIDED QUESTIONS — COMPLETED TABLE

| Gear B Radius r_B | Gear A Edge Speed $v_A = \omega_A r_A$ | Gear B Edge Speed v_B | Gear B Angular Velocity $\omega_B = v_B/r_B$ |
|------------------------|---|----------------------------|---|
| 0.5 m | 1.0 m/s | 1.0 m/s | 2.0 rad/s |
| 1.0 m | 1.0 m/s | 1.0 m/s | 1.0 rad/s |
| 2.0 m | 1.0 m/s | 1.0 m/s | 0.50 rad/s |
| 3.0 m | 1.0 m/s | 1.0 m/s | 0.33 rad/s |

Notice: v_A and v_B are always 1.0 m/s regardless of r_B . Only ω_B changes.

1 Pattern Recognition

(a) As r_B increased from 0.5 m to 3.0 m, Gear B's edge speed v_B :

Stayed the same (always 1.0 m/s — meshed gears share the same edge speed).

(b) Gear B's angular velocity ω_B :

Decreased (from 2.0 to 0.33 rad/s — bigger gears spin slower).

(c) Explain why ω_B changes even though v_B doesn't:

Since $v = \omega r$ and v is fixed by the meshing, a larger r requires a smaller ω to keep the product ωr constant.

2 Prediction

If $r_B = 0.25$ m, predict ω_B :

$$v_B = v_A = 1.0 \text{ m/s (always the same)}$$

$$\omega_B = v_B/r_B = 1.0/0.25$$

$$\omega_B = 4.0 \text{ rad/s (smaller gear = faster spin)}$$

3 The Gear Ratio

Starting from $\omega_A r_A = \omega_B r_B$, solve for ω_B/ω_A :

$$\omega_A r_A = \omega_B r_B \quad \Rightarrow \quad \frac{\omega_B}{\omega_A} = \frac{r_A}{r_B}$$

$$\frac{\omega_B}{\omega_A} = \frac{r_A}{r_B}$$

If Gear B is 3× bigger than Gear A, it spins:

$$\omega_B/\omega_A = r_A/(3r_A) = 1/3$$

1/3 as fast (one-third the angular velocity).

4 Real-World Connection — Bicycle

Pedal sprocket $r = 8.0 \text{ cm} = 0.080 \text{ m}$, rear sprocket $r = 4.0 \text{ cm} = 0.040 \text{ m}$, pedaling at 60 rpm.

(a) Chain speed (edge speed of pedal sprocket):

$$\omega_{\text{pedal}} = 60 \times 2\pi/60 = 2\pi \approx 6.28 \text{ rad/s}$$

$$v = \omega r = 6.28 \times 0.080$$

$$v \approx 0.503 \text{ m/s}$$

(b) ω of the rear sprocket:

$$\omega_{\text{rear}} = v/r_{\text{rear}} = 0.503/0.040$$

$$\omega_{\text{rear}} \approx 12.6 \text{ rad/s (= 120 rpm)}$$

(c) The rear sprocket spins:

Faster than the pedal sprocket because it is smaller.

Gear ratio: $\omega_{\text{rear}}/\omega_{\text{pedal}} = r_{\text{pedal}}/r_{\text{rear}} = 8.0/4.0 = 2$. The rear sprocket spins at exactly double the pedal rate.