

## ROTATION DAY 2: LINEAR-ANGULAR CONNECTIONS

**Warm-Up (3 min):** Two ants ride on a spinning vinyl record. Ant A is 5 cm from the center; Ant B is 15 cm from the center. The record spins at a steady rate.

- (a) Which ant completes a circle faster? \_\_\_\_\_  
 (b) Which ant travels a greater distance per revolution? \_\_\_\_\_  
 (c) Which ant has a greater speed? \_\_\_\_\_  
 (d) Both sweep the same \_\_\_\_\_ per second.

### THE BRIDGE EQUATIONS

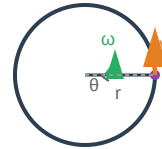
#### LINEAR-ANGULAR BRIDGE

$$v = \omega r \quad a_t = \alpha r \quad s = \theta r$$

**Tangential speed:**  $v = \omega r$

**Tangential accel:**  $a_t = \alpha r$

**Arc length:**  $s = \theta r$



**Key Insight:** These equations are the Rosetta Stone of rotation. They let you translate any angular quantity to a linear one, and vice versa. The radius  $r$  is the conversion factor.

#### **WE DO** The Record Player Revisited

A vinyl record ( $r = 15$  cm) spins at  $33\frac{1}{3}$  rpm.

**(a) Convert to rad/s:**

**(b) Linear speed at rim:**

**(c) Speed 5.0 cm from center:**

**(d) Verify ratio:**

## ROTATION DAY 2 (CONTINUED)

### YOU DO Bicycle Wheel

A bicycle wheel has radius 0.35 m and rotates at 10 rad/s.

(a) Speed of a point on the rim:

(b) Tangential acceleration ( $\alpha = 2.0 \text{ rad/s}^2$ ):

(c) Centripetal acceleration of that point:

**Warning:** Don't confuse *tangential acceleration* ( $a_t = \alpha r$ , along the edge) with *centripetal acceleration* ( $a_c = v^2/r$ , toward the center). Tangential changes speed; centripetal changes direction.

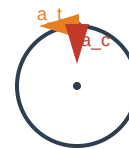
### TWO ACCELERATIONS

**Tangential:**  $a_t = \alpha r$

Speeds up or slows down.

**Centripetal:**  $a_c = v^2/r$

Turns the path (inward).



**Total:**  $a = \sqrt{a_t^2 + a_c^2}$

### QUICK Which Acceleration?

Fill in the table with Yes/No.  $a_t$  = tangential,  $a_c$  = centripetal.

Situation	$a_t = 0?$	$a_c = 0?$
Constant $\omega$ , circular path	<input style="width: 60px; height: 30px;" type="text"/>	<input style="width: 60px; height: 30px;" type="text"/>
Speeding up, circular path	<input style="width: 60px; height: 30px;" type="text"/>	<input style="width: 60px; height: 30px;" type="text"/>
Constant $v$ , straight line	<input style="width: 60px; height: 30px;" type="text"/>	<input style="width: 60px; height: 30px;" type="text"/>

## ROTATION DAY 2 (CONTINUED)

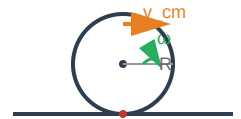
### CONSTRAINTS — WHEN OBJECTS ROLL OR CONNECT

#### ROLLING WITHOUT SLIPPING

When a wheel rolls without slipping, the contact point is momentarily at rest. This means:

$$v_{cm} = \omega R$$

The center-of-mass speed equals angular velocity times radius.



#### WE DO

 Car Tire Speed

A car tire has radius 0.32 m. The car drives at 25 m/s (about 56 mph).

**(a) Find  $\omega$  of the tire:**

**(b) Braking  $\alpha$  (5.0 s to stop):**

**(c) Revolutions while braking:**

#### YOU DO

 Pulley System

A string wraps around a pulley of radius 0.10 m. A mass hangs and drops at 2.0 m/s<sup>2</sup>.

**(a) Angular acceleration of the pulley:**

**(b) If mass drops 0.80 m from rest, radians turned:**

## EXIT TICKET & HOMEWORK

**Exit Ticket (5 min):** Two gears mesh together. Gear A has radius 10 cm; Gear B has radius 30 cm. Gear A spins at 120 rpm.

**(a) Linear speed at the edge of Gear A:**

**(b)  $\omega$  for Gear B (edges have same linear speed):**

**(c) Which gear has more angular velocity? Which edge has more linear speed?**

### HOMEWORK

#### 1. Merry-Go-Round

A 3.0 m radius merry-go-round makes one revolution in 8.0 s.

**(a) Find  $\omega$ :** \_\_\_\_\_ **(b) Speed at rim:** \_\_\_\_\_ **(c) Speed 1.0 m from center:** \_\_\_\_\_

#### 2. Centrifuge

A centrifuge tube is 12 cm from the axis. The centrifuge spins at 4000 rpm. Find the centripetal acceleration in  $\text{m/s}^2$  and as a multiple of  $g$ .

#### 3. Chain & Sprocket

On a bicycle, the pedal sprocket ( $r = 8.0$  cm) connects by chain to the rear sprocket ( $r = 4.0$  cm). The chain moves at the same linear speed at both sprockets. If you pedal at 60 rpm, what is the rear wheel sprocket's angular velocity? (Hint: same  $v = \omega r$  idea.)