ROTATION DAY 1: ANGULAR KINEMATICS — KEY

ANSWER KEY — NOT FOR DISTRIBUTION

WARM-UP Clock's Minute Hand

Minute hand length = 15 cm = 0.15 m. Halfway point at 7.5 cm = 0.075 m.

(a) Distance the tip travels in one revolution:

$$s = 2\pi r = 2\pi(0.15) = 0.30\pi$$

$s_{\text{tip}} \approx 0.94$ **m**

(b) Distance the halfway point travels:

$$s = 2\pi r = 2\pi(0.075) = 0.15\pi$$

$s_{\text{half}} \approx 0.47$ **m**

(c) Which point is moving faster?

The tip — it covers more distance in the same time, so $v_{\text{tip}} > v_{\text{half}}$.

(d) What quantity is the SAME for both?

Angular velocity ω (both sweep the same angle per second).

WE DO Convert These Angles

(a) Convert 45° to radians:

$$45^\circ \times \frac{\pi}{180^\circ} = \frac{\pi}{4}$$

$\frac{\pi}{4} \approx 0.785$ **rad**

(b) Convert $3\pi/2$ rad to degrees:

$$\frac{3\pi}{2} \times \frac{180^\circ}{\pi} = \frac{3 \times 180^\circ}{2}$$

270°

(c) How many radians in 2.5 revolutions?

$$2.5 \times 2\pi = 5\pi$$

$5\pi \approx 15.7$ **rad**

WE DO **Spinning Up a Wheel**

$\omega_0 = 0$, $\alpha = 4.0$ rad/s².

(a) ω after 3.0 s:

$$\omega = \omega_0 + \alpha t = 0 + 4.0(3.0)$$

$\omega = 12$ **rad/s**

(b) Radians turned:

$$\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2 = 0 + \tfrac{1}{2}(4.0)(3.0)^2 = 18 \text{ rad}$$

$\theta = 18$ **rad**

(c) Full revolutions:

$$\text{rev} = \frac{18}{2\pi} = 2.86$$

≈ 2.86 **revolutions**

YOU DO Braking Turntable

Spins at 33⅓ rpm, stops in 8.0 s.

(a) Convert 33⅓ rpm to rad/s:

$$33\tfrac{1}{3}\ \text{rpm} = \frac{100}{3}\ \text{rev/min} \times \frac{2\pi\ \text{rad}}{1\ \text{rev}} \times \frac{1\ \text{min}}{60\ \text{s}} = \frac{200\pi}{180} = \frac{10\pi}{9}$$

$\omega_0 \approx 3.49$ **rad/s**

(b) Find α:

$$\omega = \omega_0 + \alpha t \quad \Rightarrow \quad 0 = 3.49 + \alpha(8.0)$$

$$\alpha = -3.49/8.0$$

$\alpha \approx -0.436$ **rad/s²**

(c) Revolutions while stopping:

$$\theta = \tfrac{1}{2}(\omega_0 + \omega)t = \tfrac{1}{2}(3.49 + 0)(8.0) = 13.96\ \text{rad}$$

$$\text{rev} = 13.96/(2\pi) = 2.22$$

≈ 2.22 **revolutions (about 14.0 rad)**

QUICK Interpreting an ω-t Graph

Graph: $\omega = 5$ rad/s from 0–2 s, linear rise from 5 to 15 from 2–5 s, constant at 15 from 5–8 s.

(a) α during 0–2 s:

Slope = $\Delta\omega/\Delta t$. ω is constant at 5 rad/s.

$\alpha = 0$ **(no angular acceleration)**

(b) α during 2–5 s:

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{15 - 5}{5 - 2} = \frac{10}{3}$$

$\alpha \approx 3.33$ **rad/s²**

(c) Total angular displacement 0–8 s (area under ω-t curve):

0–2 s: rectangle = $5 \times 2 = 10$ rad

2–5 s: trapezoid = $\tfrac{1}{2}(5 + 15)(3) = 30$ rad

5–8 s: rectangle = $15 \times 3 = 45$ rad

Total $\theta = 10 + 30 + 45 = 85$ rad

EXIT TICKET & HOMEWORK

EXIT Fan Blade Acceleration

$\omega_0 = 2.0$ rad/s, $\omega = 10.0$ rad/s, $t = 4.0$ s.

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{10.0 - 2.0}{4.0} = 2.0 \text{ rad/s}^2$$

$$\theta = \tfrac{1}{2}(\omega_0 + \omega)t = \tfrac{1}{2}(2.0 + 10.0)(4.0) = 24 \text{ rad}$$

$$\text{rev} = 24/(2\pi) = 3.82$$

$\alpha = 2.0$ rad/s², $\theta = 24$ rad ≈ 3.8 revolutions

HW 1 Unit Conversion

(a) 120 rpm to rad/s:

$$120 \times \frac{2\pi}{60} = 4\pi$$

$4\pi \approx 12.6$ rad/s

(b) 15 rad/s to rpm:

$$15 \times \frac{60}{2\pi} = \frac{900}{2\pi} = \frac{450}{\pi}$$

≈ 143 rpm

(c) Car tire speed: $r = 0.32$ m, $\omega = 80$ rad/s:

$$v = \omega r = 80 \times 0.32$$

$v = 25.6$ m/s

HW 2 Centrifuge Spin-up

From rest to 12,000 rpm in 30 s.

(a) Find α:

$$\omega = 12{,}000 \times \frac{2\pi}{60} = 400\pi \approx 1257 \text{ rad/s}$$

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{400\pi}{30} = \frac{40\pi}{3}$$

$\alpha \approx 41.9$ rad/s²

(b) Revolutions during spin-up:

$$\theta = \frac{1}{2}(\omega_0 + \omega)t = \frac{1}{2}(0 + 400\pi)(30) = 6000\pi \text{ rad}$$

$$\text{rev} = 6000\pi/(2\pi) = 3000$$

$3{,}000$ **revolutions**