ANSWER KEY — ROTATION DAY 12

Full Unit Review | For teacher use only

1. QUICK RETRIEVAL — FILL THE GAPS

(a) $\tau = rF\sin\theta$

(b) I for a solid disk = $\frac{1}{2}MR^2$

(c) For rolling without slipping, $v_{\text{cm}} = \omega r$

(d) Conditions for static equilibrium: $\sum F = 0$ AND $\sum \tau = 0$

(e) Angular momentum is conserved when $\sum \tau_{\text{ext}} = 0$

(f) In the ramp race, the **solid sphere** always wins because **it has the lowest $C = I/(mr^2) = 2/5$, so the least energy goes to rotation**.

2. KINEMATICS + DYNAMICS

(a) $I = \frac{1}{2}MR^2 = \frac{1}{2}(3.0)(0.20^2) =$ **0.060 kg·m²**

(b) $\tau = FR = (15)(0.20) =$ **3.0 N·m**

(c) $\alpha = \tau/I = 3.0/0.060 =$ **50 rad/s²**

(d) $\omega = \alpha t = (50)(4.0) =$ **200 rad/s**

(e) $K = \frac{1}{2}I\omega^2 = \frac{1}{2}(0.060)(200^2) =$ **1200 J**

3. STATICS — BEAM

(a) Torques about hinge:

$T \cdot L\sin 50^\circ = M_{\text{beam}}g(L/2) + mg \cdot d$

$T(4.0)\sin 50^\circ = (20)(9.8)(2.0) + (10)(9.8)(3.0) = 392 + 294 = 686$ N·m

$T = 686/(4.0 \times 0.766) = 686/3.06 =$ **224 N**

(b) $H_x = T\cos 50^\circ = 224(0.643) =$ **144 N**

$H_y = (M+m)g - T\sin 50^\circ = 294 - 172 =$ **122 N**

4. ROLLING — SPHERE VS CYLINDER

Using $v = \sqrt{2gh/(1+C)}$ with $h = 2.5$ m:

Sphere ($C = 2/5$): $v = \sqrt{2(9.8)(2.5)/1.4} = \sqrt{35.0} =$ **5.92 m/s**

Cylinder ($C = 1/2$): $v = \sqrt{2(9.8)(2.5)/1.5} = \sqrt{32.7} =$ **5.72 m/s**

Sphere reaches the bottom first. Lower C → less energy diverted to rotation → faster translational speed. Mass and radius don't matter — only C.

5. FULL AP FRQ — PULLEY SYSTEM

(a) FBDs:

m_1: T_1 right, N up, m_1g down. m_2: T_2 up, m_2g down. Pulley: T_1 and T_2 on rim (opposite sides), pivot force at center.

(b) m_1: $T_1 = m_1a = 4.0\,a$

m_2: $m_2g - T_2 = m_2a \rightarrow 58.8 - T_2 = 6.0\,a$

(c) Pulley: $(T_2 - T_1)R = I\alpha = \frac{1}{2}MR^2 \cdot (a/R)$

$T_2 - T_1 = \frac{1}{2}Ma = \frac{1}{2}(2.0)a = 1.0\,a$

(d) Add all three equations: $m_2g = (m_1 + m_2 + \frac{1}{2}M)\,a$

$a = 58.8/(4.0 + 6.0 + 1.0) = 58.8/11.0 =$ **5.3 m/s²**

Check: $T_1 = 21.4$ N, $T_2 = 26.7$ N. $T_2 > T_1$ ✓ (net torque accelerates the pulley).

(e) The pulley's angular velocity remains constant (ignoring friction). With no string, there is no net torque on the pulley, so angular momentum is conserved and ω doesn't change.

6. COMMON MISTAKES — ERROR ANALYSIS

Mistake 1: Wrong. If the pulley has mass (and therefore rotational inertia), the string must exert different tensions on each side to produce the net torque needed to angularly accelerate the pulley. $T_1 = T_2$ is only true for a massless, frictionless pulley.

Mistake 2: Backwards reasoning. Higher rotational inertia means more energy goes INTO rotation, leaving LESS for translation. The hoop is slower, not faster. The disk (lower C) wins the ramp race.