ANSWER KEY — ROTATION DAY 11

Angular Momentum & Conservation | For teacher use only

PAGE 1 — CONSERVATION OF ANGULAR MOMENTUM

1. The Skater Spin

(a) $\omega_i = 1.5\ \text{rev/s} \times 2\pi = 3\pi \approx$ **9.42 rad/s**
(b) No external torque on the skater → L is conserved.
(c) $L_i = I_i\omega_i = (4.5)(9.42) = 42.4$ kg·m²/s. $\omega_f = L/I_f = 42.4/1.8 =$ **23.6 rad/s**
(d) $\omega_f = 23.6/(2\pi) =$ **3.75 rev/s** — that's **2.5× faster**.
(e) $K_i = \frac{1}{2}(4.5)(9.42^2) \approx 200$ J. $K_f = \frac{1}{2}(1.8)(23.6^2) \approx 500$ J. KE **increased by factor 2.5**. Extra energy came from **muscular work done pulling her arms inward**.

2. Merry-Go-Round + Person

(a) $I_{\text{disk}} = \frac{1}{2}MR^2 = \frac{1}{2}(150)(2.0^2) =$ **300 kg·m²**
(b) $I_{\text{person}} = mR^2 = (60)(2.0^2) =$ **240 kg·m²**
(c) $I_{\text{total}} = 300 + 240 =$ **540 kg·m²**
(d) $\omega_f = I_i\omega_i/I_f = (300)(0.50)/540 =$ **0.28 rad/s**
(e) $K_i = \frac{1}{2}(300)(0.50^2) = 37.5$ J. $K_f = \frac{1}{2}(540)(0.278^2) = 20.8$ J. **KE is NOT conserved** — lost 16.7 J, just like an inelastic collision.

3. Catch on a Turntable

(a) $L_{\text{ball}} = mvr = (0.50)(10)(0.80) =$ **4.0 kg·m²/s**
(b) $I_{\text{total}} = 5.0 + (0.50)(0.80^2) = 5.0 + 0.32 =$ **5.32 kg·m²**
(c) $\omega_f = 4.0/5.32 =$ **0.75 rad/s**

PAGE 2 — ANGULAR MOMENTUM AS A VECTOR

4. Bike Wheel Gyroscope Demo

(a) $\vec{L}$ points **along the axle (horizontal)**, direction given by right-hand rule for the spin.
(b) $\vec{\tau} = \vec{r} \times \vec{F}$ is **horizontal, perpendicular to axle** (into the page if $\vec{L}$ points right and $m\vec{g}$ points down).
(c) Since $\Delta\vec{L}$ is horizontal, $\vec{L}$ rotates **sideways (precesses)** — it does NOT tilt down.
(d) **Decreases.** $\Omega_{\text{prec}} = \tau/L = mgr/(I\omega)$. Larger ω → larger L → slower precession.
(e) It **falls straight down**. No angular momentum → no gyroscopic effect.

PAGE 3 — POINT MASS L & HELICOPTER

5. Ball Past a Pivot

(a) $L = mvr_\perp = (0.50)(8.0)(1.2) =$ **4.8 kg·m²/s**
(b) **No** — L is constant along the entire path (no external torques about O). As r increases, the component of v perpendicular to r decreases, keeping $L = mvr\sin\theta$ constant.
(c) **Into the page** (right-hand rule: $\vec{r}$ points upward from O toward the ball, $\vec{v}$ points right → curl from $\vec{r}$ toward $\vec{v}$ gives a clockwise curl, thumb points into page). Equivalently: the ball would orbit O clockwise, so by the spin right-hand rule, $\vec{L}$ is into the page.

6. The Helicopter Problem

(a) By Newton's 3rd Law, the engine torque on the rotor produces an **equal and opposite reaction torque on the body → the fuselage would spin counterclockwise** (opposite to CW rotor).
(b) The tail rotor pushes air sideways, creating a **force at a distance from the helicopter's vertical axis → a torque that counteracts the reaction torque**, preventing the body from spinning.

PAGE 4 — ANGULAR IMPULSE

7. Conceptual Checks — True/False

(a) FALSE. $L = I\omega$ is constant, but if I changes (e.g., skater pulling arms in), ω changes. Zero torque means constant L, not necessarily constant ω.
(b) FALSE. Her angular momentum stays the same (conserved, no external torque). It's ω that increases because I decreases.
(c) FALSE. Angular momentum is only conserved when $\sum \tau_{\text{ext}} = 0$. If there is a net external torque, L changes.
(d) TRUE. $L = mvr_\perp$ about any point not on the line of motion.

8. The High Dive

Tuck: ω: **increases** | L: **stays the same**
Layout: ω: **decreases** | L: **stays the same**
Key check: **No, L did not change.** No external torque acts on the diver during airtime (gravity acts at the center of mass, producing no torque about the CM).

9. Spinning Platform with Weights

(a) $I_i = 4.0 + 2(3.0)(0.80^2) = 4.0 + 3.84 =$ **7.84 kg·m²**
(b) $I_f = 4.0 + 2(3.0)(0.20^2) = 4.0 + 0.24 =$ **4.24 kg·m²**
(c) $\omega_f = I_i\omega_i / I_f = (7.84)(2.0)/4.24 =$ **3.7 rad/s**
(d) $K_i = \frac{1}{2}(7.84)(2.0^2) = 15.7$ J. $K_f = \frac{1}{2}(4.24)(3.70^2) = 29.0$ J. Factor = **1.85× (KE nearly doubled)**. Energy came from the person's muscles doing work pulling the masses inward.

PAGE 5 — EXIT TICKET & HOMEWORK

10. Exit Ticket — Satellite

The 20 kg of panels start stowed inside the body, so initially everything spins as one sphere of total mass 220 kg.
$I_i = \frac{2}{5}(220)(0.50^2) = 22$ kg·m²
After extending: $I_f = \frac{2}{5}(200)(0.50^2) + (20)(3.0^2) = 20 + 180 = 200$ kg·m²
$L_i = (22)(2.0) = 44$ kg·m²/s → $\omega_f = 44/200 =$ **0.22 rad/s**

HW 1. Basic Conservation

$I_{\text{total}} = 0.40 + 0.40 = 0.80$ kg·m². $L = (0.40)(8.0) = 3.2$ kg·m²/s.
$\omega_f = 3.2/0.80 =$ **4.0 rad/s**

HW 2. Neutron Star

Step 1 — Convert period to angular velocity:
$T_1 = 30 \text{ days} = 30 \times 86{,}400 = 2.592 \times 10^6$ s

$$\omega_1 = \frac{2\pi}{T_1} = \frac{2\pi}{2.592 \times 10^6} = 2.42 \times 10^{-6} \text{ rad/s}$$

Step 2 — Apply conservation of angular momentum:

$I_1\omega_1 = I_2\omega_2$. Since $I = \frac{2}{5}MR^2$ and mass is conserved:

$\frac{2}{5}MR_1^2\,\omega_1 = \frac{2}{5}MR_2^2\,\omega_2 \rightarrow$ cancel $\frac{2}{5}M$: $\rightarrow R_1^2\omega_1 = R_2^2\omega_2$

$$\omega_2 = \omega_1 \times \frac{R_1^2}{R_2^2} = (2.42 \times 10^{-6}) \times \frac{(700{,}000)^2}{(10)^2} = (2.42 \times 10^{-6})(4.9 \times 10^9)$$

$\omega_2 =$ **11,900 rad/s**

Step 3 — Convert back to period:

$$T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{11{,}900} = 5.3 \times 10^{-4}\ \text{s} \approx 0.53\ \text{ms}$$

This is realistic — millisecond pulsars spin hundreds of times per second!

HW 3. Angular Impulse

(a) $\Delta L = \tau\,\Delta t = (12)(4.0) =$ **48 kg·m²/s**

(b) $\omega_f = \Delta L / I = 48/3.0 =$ **16 rad/s**

HW 4. AP Comparison (Sample response)

Linear momentum $p = mv$ is conserved when $\sum F_{\text{ext}} = 0$. Angular momentum $L = I\omega$ is conserved when $\sum \tau_{\text{ext}} = 0$.

Yes, one can be conserved while the other isn't.

Example: A ball on a string in a vertical circle — the tension and gravity change $\vec{p}$, but tension passes through the center and produces no torque about it, so L about the center is conserved while $\vec{p}$ is not.

Reverse example: A person jumping onto a merry-go-round — linear momentum of the system can be conserved (no net external horizontal force) while angular momentum about any external point changes due to external torques from the axle.