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ROTATION DAY 10: ROLLING MOTION

Warm-Up (3 min): A box sits on a table. You push near the top edge. Sometimes it slides, sometimes it tips over.

1. **(a)** What determines whether the box slides or tips? (Think about the forces acting on it).
2. **(b)** Now imagine a cylinder on a table. You push it. Does it ever slide *without* rotating? Why is a circle fundamentally different from a box?

WHY THINGS ROLL: CONSTANT TIPPING

THE BLOCK

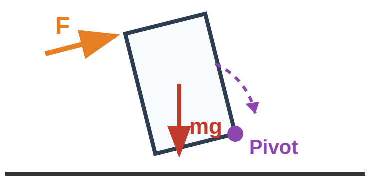
A block only tips when the torque from your push *exceeds* the restoring torque from gravity. The pivot point is the front corner.

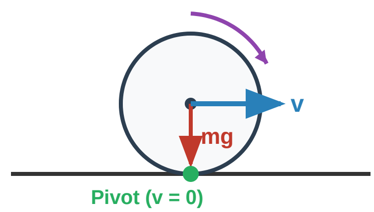
Push high → more applied torque → it tips. Push low → it just slides.

THE CIRCLE

A circle tips with **any** net force, because the pivot (contact point) is always at the very bottom edge.

Because the center of mass is directly over the pivot, gravity provides *zero* restoring torque. Rolling is just **constant tipping** over a moving pivot.





The Rolling Constraint: $v_{cm} = \omega R$ and $a_{cm} = \alpha R$.
If $v_{cm} > \omega R$, the object is sliding (like a bowling ball right after release). If $v_{cm} < \omega R$, it's spinning in place (like tires stuck on ice). Rolling without slipping is the perfect balance between the two.

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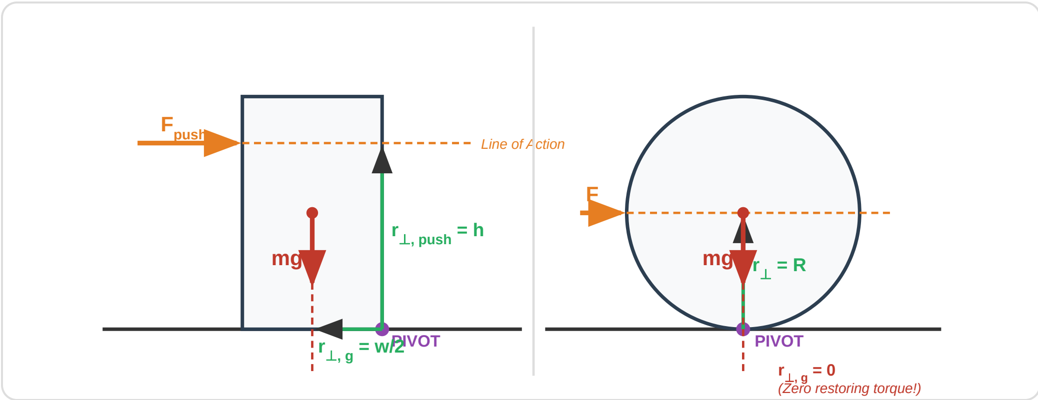
DEEP DIVE: LINE OF ACTION & TIPPING

LINE OF ACTION & LEVER ARM

To determine whether an object tips or rolls, extend any force vector into a dashed **Line of Action**. The **lever arm ($r_\perp$)** is the shortest perpendicular distance from the pivot to that line.

$$\tau = F \cdot r_\perp$$



WE DO **The Tipping Threshold**

Using the diagrams above, let's write the mathematical rules for tipping vs. rolling.

1. **(a) The Box:** In order for the box to tip to the right, what must be true about the applied torque vs. the gravitational torque? Write the equation using F, h, m, g, and w.

2. **(b) The Circle:** What is the magnitude of the restoring torque from gravity for the circle? Use this to explain why a circle will rotate forward if *any* force $F > 0$ is applied.

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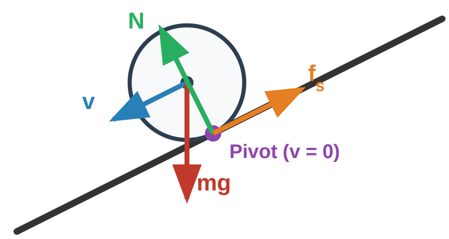
THE PARADOX: DOES THE RAMP DO WORK?

ZERO WORK AT THE CONTACT POINT

A ball rolls down a ramp and speeds up. The ramp pushes on the ball with two forces: **normal force** and **static friction**. (Yes — rolling friction *is* static friction! The contact point doesn't slide.)

Both forces act at the **contact point**. Because it rolls without slipping, the contact point is momentarily at rest ($v = 0$).

Since $W = F \cdot \Delta x$ and $\Delta x = 0$, the ramp does **zero mechanical work**. It just redirects gravitational energy into rotational KE! Mechanical energy is conserved.



THE MASTER SHAPE TABLE

From Yesterday: We used energy conservation to derive $v = \sqrt{\frac{2gh}{1+c}}$ for an object with $I = cMR^2$. Today we map out every shape to build a reference table you'll use for the rest of the unit.

WE DO **Complete the Table**

Calculate what fraction of total KE is rotational vs. translational for each shape.

Hint: $K_{rot}/K_{total} = c/(1+c)$ *and* $K_{trans}/K_{total} = 1/(1+c)$.

Shape	$I = cMR^2$	c	Speed at bottom	% Rotational KE	% Trans. KE	Rank
Solid sphere	$\frac{2}{5}MR^2$	$\frac{2}{5}$				
Solid cylinder	$\frac{1}{2}MR^2$	$\frac{1}{2}$				
Thin hoop / ring	$1MR^2$	1				
Sliding block	—	0		0%	100%	

YOU DO **Pattern Recognition**

1. **(a)** Which shape "wastes" the most energy on spinning? Which wastes the least?
2. **(b)** A solid disk *always* splits its KE as ___% translational, ___% rotational. Memorize this!
3. **(c)** Would a hollow sphere ($c = \frac{2}{3}$) finish faster or slower than a solid sphere? Faster or slower than a hoop?

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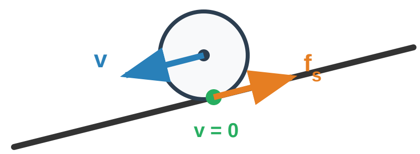
WHEN ENERGY WORKS — AND WHEN IT DOESN'T

SCENARIO A: ROLLING WITHOUT SLIPPING (ENERGY WORKS!)

STATIC FRICTION = ENERGY CONSERVED

Because the contact point is at rest, static friction does no work. The object's gravitational potential energy is perfectly converted into $K_{trans} + K_{rot}$.

Use $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$



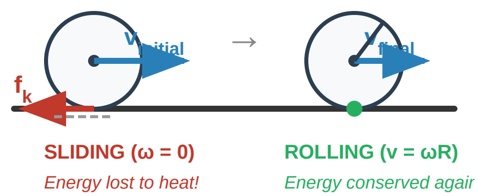
SCENARIO B: SLIDING THEN ROLLING (ENERGY LOST!)

THE BOWLING BALL PROBLEM

A bowling ball is thrown with v_{cm} but **no spin** ($\omega = 0$). It skids on the lane.

Because it is slipping, it uses **kinetic friction**. The contact point is rubbing against the floor ($\Delta x \neq 0$), so kinetic friction does work and grinds away energy as heat.

Eventually, friction slows v_{cm} and speeds up ω until $v = \omega R$. At that exact moment, it starts rolling and energy is conserved again!



QUICK **Which Scenario?**

Situation	Friction type	Energy conserved?
Ball rolls down a rough ramp from rest		
Bowling ball thrown with no spin on a lane		
Ball rolling on a perfectly frictionless surface		

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PRACTICE

DECISION TREE: ROLLING PROBLEMS

1. **Is it rolling without slipping?** → Use $v_{cm} = \omega R$ and energy conservation.
2. **Is it sliding?** → Kinetic friction does work. Energy is NOT conserved. Use forces + torque.
3. **What shape?** → Look up c in the shape table. Use $v = \sqrt{2gh/(1+c)}$ for ramp problems.

HOMEWORK

HW 1 Energy Split Shortcut

A solid cylinder rolls at $v_{cm} = 6.0$ m/s. Without calculating the actual energies, use the KE fraction shortcut from the shape table to find the translational and rotational KE if the cylinder's mass is 2.0 kg.

HW 2 The Bowling Ball

A bowling ball (solid sphere, mass 6.0 kg, radius 0.11 m) is thrown at 8.0 m/s with no spin onto a lane with $\mu_k = 0.10$.

1. **(a)** What specific force causes the ball's center of mass to slow down?
2. **(b)** What specific force causes the ball to start spinning?
3. **(c)** Challenge: When $v_{cm} = \omega R$, the ball stops sliding. Use $f_k = Ma_{cm}$ and $\tau = I\alpha$ to show that the final rolling speed is $\frac{5}{7}v_0$. Where did the lost energy go?

HW 3 Will It Tip or Will It Slide?

A uniform box is 0.40 m wide and 1.0 m tall, with mass 20 kg. You push horizontally at the very top edge. The coefficient of static friction between the box and the floor is $\mu_s = 0.35$.

1. **(a)** What is the minimum force needed to make the box *slide*? (Hint: when does your push overcome static friction?)
2. **(b)** What is the minimum force needed to make the box *tip*? (Hint: set $\tau_{push} = \tau_{gravity}$ about the pivot corner.)
3. **(c)** Which happens first — tipping or sliding? How would your answer change if you pushed at the midpoint instead of the top?