NAME: _____ AP PHYSICS 1 DATE: _____

MOMENTUM DAY 6: CENTER OF MASS + 2D MOMENTUM

Warm-Up (3 min): Direction Check

A cart has momentum $+12\,\text{kg·m/s}$ in x and $-5\,\text{kg·m/s}$ in y.

1) Which quadrant is the momentum vector in? _____

2) Is the direction above or below +x? _____

CENTER OF MASS (C.O.M.)

THE BIG IDEA

The **center of mass** is the "balance point" of a system. Its velocity depends on total momentum:

$$\vec{v}_{cm} = \frac{\vec{p}_{total}}{m_{total}} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + \cdots}{m_1 + m_2 + \cdots}$$

Key insight: If no external forces act, $\vec{v}_{cm}$ stays constant—even during collisions and explosions!

1 C.O.M. Velocity: Explosion

A 2.0 kg cart and 3.0 kg cart are at rest on a frictionless track. A spring between them releases, pushing them apart. Cart A moves left at 6.0 m/s.

a) What is v_{cm} before the explosion?

b) What is v_{cm} after? Why?

The Projectile Trap: If a firework is launched and explodes at the peak of its flight, the fragments fly everywhere. However, the **Center of Mass** continues to follow the original parabolic path (as if it never exploded) because gravity is an external force acting on the whole system.

2D MOMENTUM STRATEGY

2D = TWO 1D PROBLEMS

In 2D, split momentum conservation into components:

$$\sum p_{x,i} = \sum p_{x,f} \quad \textbf{and} \quad \sum p_{y,i} = \sum p_{y,f}$$

Tip: Pick your axes wisely! Align x with the initial velocity if possible.

WARM-UP: VECTOR COMPONENTS

2 Break a Velocity Vector into Components

A puck moves at $v = 6.0\,\text{m/s}$ at 40° above the +x axis. Find v_x and v_y.

$v_x = v\cos\theta =$ _____

$v_y = v\sin\theta =$ _____

Tip: If θ is from +x, then x uses cos, y uses sin.

2D INELASTIC COLLISION (STICK)

3 Two Pucks, Perpendicular Velocities

Puck 1 ($m_1 = 0.50\,\text{kg}$) moves east at $v_1 = 4.0\,\text{m/s}$. Puck 2 ($m_2 = 0.75\,\text{kg}$) moves north at $v_2 = 3.0\,\text{m/s}$. They collide and stick. Find the final speed and direction of the combined mass.

Momentum before (components):

$p_{x,i} = m_1 v_1 + 0 =$ _____

$p_{y,i} = 0 + m_2 v_2 =$ _____

After collision: total mass $m_T = m_1 + m_2$

$p_{x,f} = m_T v_f \cos\theta$

$p_{y,f} = m_T v_f \sin\theta$

Solve: Find v_f and θ (angle above +x).

Useful relationships: $v_f = \sqrt{v_{fx}^2 + v_{fy}^2}$, and $\tan\theta = \frac{v_{fy}}{v_{fx}}$.

YOU DO: 2D MOMENTUM PRACTICE

4 Unknown Speed from Momentum Components

A 0.20 kg puck moves at 5.0 m/s east. It collides with a 0.30 kg puck moving south at an unknown speed v. They stick and move at 20° south of east with a speed of 2.1 m/s.

a. Write the x-component momentum equation.

b. Write the y-component momentum equation.

c. Solve for v (the initial speed of the 0.30 kg puck).

5 Conceptual: Direction of Final Motion

Two objects stick together in a collision. Object A has momentum 8 kg·m/s east. Object B has momentum 6 kg·m/s north.

a. Without calculating a speed, what quadrant is the final momentum in?

b. Sketch the momentum vector addition.

6 Reverse It: 2D Explosion

An object at rest explodes into 3 pieces. Piece A flies North. Piece B flies East. In which quadrant must Piece C fly to conserve momentum?

Hint: The total momentum must remain zero.

FRQ SYNTHESIS SCAFFOLD

FRQ Glancing Collision + Justification

A 0.40 kg puck slides on frictionless ice at 6.0 m/s east and collides with a 0.60 kg puck moving north at 2.0 m/s. After the collision, they stick and move together.

a. Define an appropriate system for using momentum conservation.

b. Write the momentum conservation equations for x and y.

c. Calculate the magnitude of the final velocity.

d. Calculate the direction of the final velocity (angle from +x).

e. Explain why momentum is conserved for your system during the collision.

f. Compare K_{before} and K_{after}. Is kinetic energy conserved? Explain.

g. **Synthesis:** Without doing calculation, describe the velocity vector of the **center of mass** of the two-puck system *after* the collision compared to *before*. Justify your answer.

Unit Summary: We have covered Impulse, 1D Collisions, Bouncing vs. Sticking, Energy Loss, and 2D Vector Momentum. Tomorrow is Review!