FLUIDS DAY 3: FLOW & BERNOULLI'S EQUATION

Physics in the Wild: Why Planes Fly & Balls Curve

Airplane wings, curveballs, and the shower curtain blowing inward all involve pressure differences in moving air. Bernoulli is one useful tool, but the first expert move is deciding whether its assumptions actually apply.

Emergence:

"Bernoulli's equation is energy conservation in disguise. The reductionist approach — tracking each of the 10^{26} molecules in a glass of water — is hopeless. But when we zoom out and describe the fluid as a continuous medium, a beautiful equation emerges: $P + \frac{1}{2}\rho v^2 + \rho g y$ = constant. We don't need to know what every molecule is doing. The emergent description is not just simpler — it's the only one that's practical."

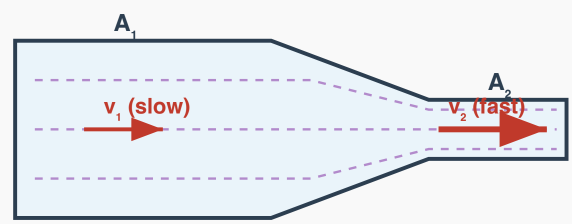
CONTINUITY: WHAT FLOWS IN MUST FLOW OUT

EQUATION OF CONTINUITY

$$A_1 v_1 = A_2 v_2$$

Incompressible fluid: flow rate $Q = Av$ is constant.

Narrow pipe → **faster** flow. Wide pipe → **slower** flow.



WE DO Exercise 1: Narrowing Pipe

A water pipe narrows from A = 4.0×10^{-3} m² to A = 1.0×10^{-3} m². Water enters at 2.0 m/s. Find the speed in the narrow section.

YOU DO Exercise 2: Blood in Capillaries

Blood flows through the aorta (r = 1.0 cm) at 0.30 m/s. Into a capillary (r = 4.0×10^{-6} m). If all blood went through one capillary, how fast would it flow? (You have billions — that's why!)

BERNOULLI'S EQUATION

ENERGY CONSERVATION FOR FLUIDS

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2$$

Units: Pa = J/m³. Each term is energy per unit volume: P (pressure), $\tfrac{1}{2}\rho v^2$ (kinetic), $\rho g y$ (potential).

Key insight: along one steady, ideal streamline at the same height, greater speed means lower pressure.

Energy Conservation: KE + PE + "pressure energy" = constant along a streamline. If height is unchanged and the same fluid parcel speeds up, pressure drops.

WE DO **Exercise 3: Horizontal Pipe**

Horizontal pipe. Point 1: v_1 = 2.0 m/s, P_1 = 250,000 Pa. Point 2 (narrower): v_2 = 6.0 m/s. Find P_2. ($y_1 = y_2$ → gravity cancels.)

SYNTHESIS **Exercise 4: Wing Lift — Two Stories**

An airplane wing has area A = 12 m². Air above the upper surface moves at $\mathbf{v_{top}}$ **= 90 m/s**; air along the lower surface at $\mathbf{v_{bot}}$ **= 70 m/s**. ρ_{air} = 1.2 kg/m³.

(a) Naive Bernoulli. Pretend the top and bottom are on the same streamline. Compute ΔP and the predicted lift F = ΔP·A. Then state two reasons this calculation should make you suspicious.

(b) Streamline curvature. A fluid parcel following the curved upper-surface flow needs centripetal force. Where can that force come from in a fluid? Use this to argue why pressure must drop as you move down through the curved flow toward the upper wing surface.

(c) Compare. Both methods predict lower pressure on top. What does each method capture? What does each miss?

When "fast = lower pressure" fails: Bernoulli's equation holds along a single streamline of steady, ideal flow. The classic "blow across a paper" demo and the lazy "air-over-a-wing because it travels faster" story compare different streamlines. Before using Bernoulli, ask: am I staying on one streamline, or sneaking between two?

TORRICELLI'S THEOREM: DRAINING A TANK

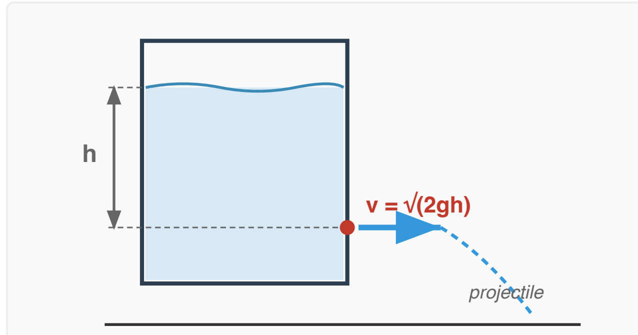
SPECIAL CASE OF BERNOULLI'S

Open tank, hole in side: both surface and hole are at atmospheric pressure (P cancels). Surface barely moves ($v_1 \approx 0$). Bernoulli collapses to:

$$v = \sqrt{2gh}$$

where h = depth of hole below water surface.

Same as an object dropped from height h — both are energy conservation.



WE DO **Exercise 5: Tank Drainage**

Water exits a hole 2.0 m below the surface of a large tank. Find the exit speed.

SKILL **Exercise 6: Torricelli + Projectile Motion**

Tank on a table; hole drilled 0.80 m below the water surface, 1.2 m above the floor. **(a)** Exit velocity? **(b)** Time to fall to floor? **(c)** Horizontal range?

Tomorrow's Lab + Synthesis: You'll test Torricelli's theorem, then use pressure, buoyancy, continuity, and Bernoulli together before the quiz.

EXIT TICKET & HOMEWORK

EXIT **Garden Hose**

Water flows through a garden hose (diameter 2.0 cm) at 1.5 m/s. You place your thumb over the end, reducing the opening to 0.50 cm diameter. **(a)** Exit speed? **(b)** Pressure difference between hose interior and exit? **(c)** Why does it shoot farther?

HOMEWORK PROBLEMS

1 Vertical Pipe

A pipe carries water from the basement ($y = 0$, $v = 1.0$ m/s, $P = 3.0 \times 10^5$ Pa) to the second floor ($y = 5.0$ m) where the pipe narrows so $v = 3.0$ m/s. Find the pressure on the second floor.

2 Tank Drainage & Projectiles

A large water tank has a hole 3.0 m below the waterline and 1.5 m above the ground. **(a)** Exit speed? **(b)** Horizontal range?

3 Pitot Tube

An aircraft's pitot tube measures a stagnation pressure of 102,500 Pa. The static pressure at that altitude is 100,000 Pa. Air density is 1.1 kg/m³. Find the airspeed. (Hint: $\frac{1}{2}\rho v^2 = P_{stag} - P_{static}$)

Tomorrow: Short Torricelli lab, synthesis problems, and quiz prep.