ROTATING SYSTEMS DAY 5: ANGULAR MOMENTUM WITH CALCULUS

WARMUP **Day 4 Bridge**

A disk with $I = 2.0$ kg·m² spins at $\omega = 6.0$ rad/s. Compute its angular momentum $L = I\omega$, with units.

The third form of Newton's second law. This year it has appeared as $F = ma$, then $F = dp/dt$ (Unit 4), and now $\boldsymbol{\tau = dL/dt}$ — torque is the rate of change of angular momentum. Yesterday's conservation law is its zero-torque special case. Today we run the calculus directly.

DIFFERENTIATE THE ANGULAR MOMENTUM

Given $L(t)$, the net torque at any instant is its slope:

$$\tau = dL/dt$$

Units check: (kg·m²/s)/s = kg·m²/s² = N·m. For a rigid body with CONSTANT I, the chain $L = I\omega$ differentiates to $\tau = I(d\omega/dt) = I\alpha$ — Day 9 of the last unit falls out as a special case. When I itself changes (arms in, arms out), $\tau = dL/dt$ is the version that stays true.

WE DO **Exercise 1: Slope of L(t)**

A flywheel's angular momentum grows as $L(t) = 4t^2 + 2t$ (kg·m²/s when t is in seconds). **(a)** Find $\tau(t) = dL/dt$. **(b)** Evaluate the net torque at $t = 2.0$ s. **(c)** Is the torque constant here? How can you tell from $L(t)$ without differentiating?

ANGULAR IMPULSE: ACCUMULATING TORQUE OVER TIME

THE LAST ROW OF THE ACCUMULATION TABLE

Run $\tau = dL/dt$ backwards and torque accumulates into angular momentum:

$$\int \tau \, dt = \Delta L$$

One strip: (N·m)(s) = kg·m²/s — a parcel of angular momentum. This is the rotational twin of Unit 4's impulse integral $\int F \, dt = \Delta p$, and the same graph skills apply: rectangles, triangles, and antiderivatives under a τ-t curve.

WE DO **Exercise 2: Ramp It Up**

A motor's torque on a turbine ramps as $\tau(t) = 12t$ (N·m when t is in seconds) from $t = 0$ to $t = 3.0$ s. **(a)** Find the angular impulse delivered, by integral. **(b)** The turbine has $I = 9.0$ kg·m² and starts at rest — find its final angular velocity.

YOU DO **Exercise 3: Piecewise Torque**

A torque holds constant at 8.0 N·m for 2.0 s, then ramps in a straight line down to 0 over the next 1.0 s. **(a)** Sketch the τ-t graph. **(b)** Find the total ΔL — rectangle plus triangle, units attached.

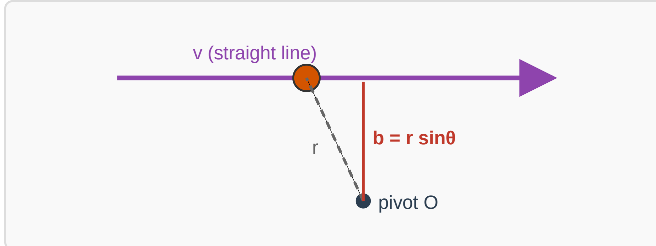
A PARTICLE HAS ANGULAR MOMENTUM TOO

L = MVR SINΘ — THE CROSS PRODUCT PAYS OFF

A moving particle carries angular momentum about ANY chosen point:

$$\mathbf{L = mvr\ sin\theta = \|r \times p\|}$$

The useful reading: r sinθ is the **closest-approach distance** b between the point and the particle's line of motion. So L = mvb — and for a particle cruising in a straight line at constant speed, b never changes: **L is constant without any force at all.** That is why a ball flying past a door hinge can still slam the door.



WE DO **Exercise 4: Flyby**

A 2.0 kg ball moves in a straight line at 3.0 m/s. Its line of motion passes 4.0 m from point O at closest approach. **(a)** Find the ball's angular momentum about O. **(b)** A moment later the ball is farther away, with r sinθ still equal to 4.0 m — what is L now, and why?

CALC **Exercise 5: The Skater, Justified**

A skater spins with arms out: I = 4.0 kg·m², ω = 3.0 rad/s. She pulls her arms in to I = 2.0 kg·m². **(a)** Which quantity is conserved, and which single equation from today GUARANTEES it? **(b)** Find her new angular velocity.

PAPER PRACTICE

PRACTICE PROBLEMS

P1 Slope of a Cubic

A rotor's angular momentum follows $L(t) = 3t^3$ (kg·m²/s when t is in seconds). Find the net torque on it at t = 2.0 s.

P2 Accumulate, Then Convert

A torque ramps as $\tau(t) = 6t$ (N·m when t is in seconds) for 4.0 s on a wheel with $I = 12$ kg·m², initially at rest. **(a)** Find ΔL by integral — units of one strip first. **(b)** Find the final angular velocity.

P3 Flyby, Your Turn

A 0.50 kg puck slides in a straight line at 8.0 m/s, passing 2.0 m from point O at closest approach. Find its angular momentum about O.

P4 When I Changes Too

For a rigid body, $\tau = dL/dt$ with $L = I\omega$. **(a)** If I is constant, differentiate to recover last unit's $\tau = I\alpha$. **(b)** Product-rule preview (read and respond): when I ALSO changes, $dL/dt = I(d\omega/dt) + \omega(dI/dt)$. For the arms-in skater with $\tau = 0$, which term must be negative while the other is positive — and what does each term describe physically?