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ROTATING SYSTEMS DAY 4: ANGULAR MOMENTUM

$$\text{Rigid body about a fixed axis: } L = I\omega \qquad \text{If } \sum \tau_{\text{ext}} = 0: \quad L_i = L_f$$

Later today: a moving particle can also have angular momentum about a point, with $L = mvr_\perp$.

CONSERVATION OF ANGULAR MOMENTUM

WE DO The Skater Spin

A figure skater has $I = 4.5$ kg·m² with arms extended and spins at 1.5 rev/s. She pulls her arms in, reducing I to 1.8 kg·m².

(a) Convert ω_i to rad/s.

(b) Is there an external torque? → L is conserved.

(c) Find ω_f.

(d) Convert ω_f to rev/s. How many times faster?

(e) Did her kinetic energy change? Calculate K_i and K_f. Where did the extra energy come from?

COLLISIONS ON A TURNTABLE

WE DO Merry-Go-Round + Person

A 150 kg merry-go-round (solid disk, radius 2.0 m) spins at 0.50 rad/s. A 60 kg person jumps onto the edge from rest (radially).

(a) Find $I_{\text{disk}} = \frac{1}{2}MR^2 =$ ________

(b) Find $I_{\text{person}} = mR^2$ (treat as point mass) = ________

(c) $I_{\text{total after}} = I_{\text{disk}} + I_{\text{person}} =$ ________

(d) Apply conservation: $I_i\omega_i = I_f\omega_f$. Find ω_f.

(e) Is kinetic energy conserved? Calculate both. This is like an inelastic collision!

YOU DO Catch on a Turntable

A person stands at the center of a stationary turntable ($I_{\text{system}} = 5.0$ kg·m²). A friend throws a ball (mass 0.50 kg) horizontally. The ball lands in the person's hands at $r = 0.80$ m from the axis, moving at 10 m/s tangentially.

(a) What is the ball's angular momentum about the axis before being caught? (Hint: $L = mvr$ for tangential motion.)

(b) After the catch, what is the total I of the system (turntable + person + ball)?

(c) Find ω after the catch.

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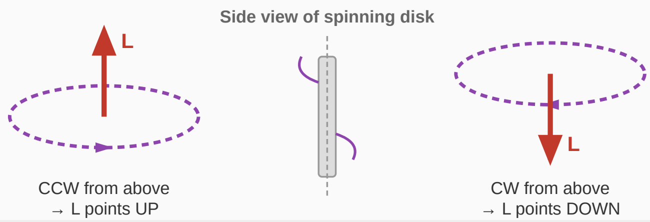
ANGULAR MOMENTUM IS A VECTOR!

DIRECTION OF $\vec{L}$

So far we've treated L as a number — positive or negative. But angular momentum is a **vector**. Its direction is along the **axis of rotation**, determined by the **right-hand rule**: curl your fingers in the direction of spin, and your thumb points in the direction of $\vec{L}$.

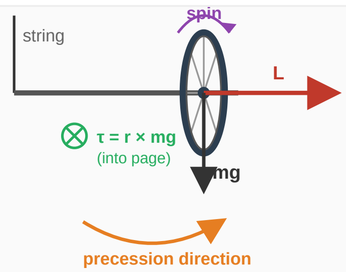


Key insight: Since $\vec{L}$ is a vector, changing its *direction* — not just its magnitude — requires a net external torque. The relationship $\vec{\tau}_{\text{net}} = \frac{\Delta \vec{L}}{\Delta t}$ tells us that torque applied over time changes angular momentum. **This takes torque × time, not work!**

DEMO: THE BIKE WHEEL GYROSCOPE

DEMO **Spinning Bike Wheel on a String**

Your teacher holds a spinning bike wheel by one end of its axle and releases the other end. Instead of tipping straight down right away, the wheel begins to rotate (precess) horizontally.



What you observe:

(a) Which direction does $\vec{L}$ point when the wheel spins? (Use the right-hand rule.)

(b) Gravity pulls the free end down. What direction is the torque $\vec{\tau} = \vec{r} \times \vec{F}$? (Hint: it's perpendicular to both the axle and gravity — it's *horizontal*.)

(c) Since $\Delta \vec{L} = \vec{\tau}\,\Delta t$ is horizontal, does $\vec{L}$ tilt *down* or rotate *sideways*?

Why doesn't it immediately tip straight down? In this idealized model, the torque from gravity is *perpendicular* to $\vec{L}$, so it mainly changes the *direction* of $\vec{L}$ rather than its magnitude — just like a centripetal force changes the direction of velocity without changing speed. The wheel turns sideways (precesses) instead of just dropping straight down right away.

THINK **Predictions**

(d) If you spin the wheel *faster*, the precession rate: increases / decreases / stays the same

Explain: (Hint: bigger $|\vec{L}|$ means the same torque causes a *smaller* angular change per second.)

(e) What happens if the wheel is *not* spinning when you release one end? Why?

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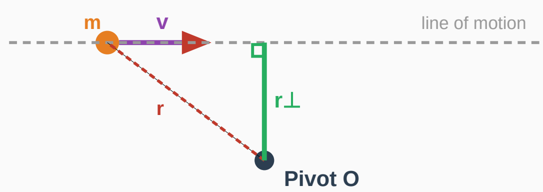
L OF A POINT MASS IN A STRAIGHT LINE

The same angular momentum idea also works when an object is *not* spinning as a rigid body. A particle doesn't have to move in a circle to have angular momentum about a point! For any particle with momentum $p = mv$:

$$L = mvr_\perp$$

where $r_\perp$ is the **perpendicular distance** from the axis to the particle's line of motion (the "closest approach distance").



Equivalently: $L = mvr\sin\theta$, where θ is the angle between $\vec{r}$ and $\vec{v}$. When motion is tangential, $\sin\theta = 1$. When the particle moves straight toward or away from the axis, $\sin\theta = 0$ and $L = 0$.

Big picture: An object's total angular momentum = *orbital* L (motion of its center of mass about the axis) + *spin* L (rotation about its own center). Think: Earth orbits the Sun **and** spins on its axis.

YOU DO Ball Past a Pivot

A 0.50 kg ball travels at 8.0 m/s in a straight line that passes 1.2 m from point O.

(a) $L = mvr_\perp =$ _____ kg·m²/s **(b)** Does L change along the path (no external torques)?

____

(c) Using the right-hand rule, is $\vec{L}$ into or out of the page? ____

ENGINEER The Helicopter Problem

The engine applies a torque to spin the main rotor blades **clockwise** (as seen from above).

(a) By Newton's 3rd Law, what must happen to the *body* of the helicopter?

(b) The small tail rotor pushes air sideways. Explain its purpose using the concept of torque.

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ANGULAR IMPULSE

ANGULAR IMPULSE–MOMENTUM THEOREM

Just as $F\Delta t = \Delta p$: $\tau\Delta t = \Delta L = I_f\omega_f - I_i\omega_i$ — a torque applied for a time changes angular momentum.

QUICK Conceptual Checks True/False. Correct any false statements.

(a) If net external torque is zero, angular velocity must be constant. ___ **(b)** A figure skater's angular momentum increases when she pulls her arms in. ___

(c) Angular momentum is always conserved. ___ **(d)** A ball moving in a straight line can have angular momentum about a point not on the line. ___

SPORT The High Dive

A diver leaves the board with angular momentum L. Air resistance is negligible.

Tuck: She grabs her knees. I decreases.

ω: increases / decreases / stays the same

L: increases / decreases / stays the same

Layout: She straightens out. I increases.

ω: increases / decreases / stays the same

L: increases / decreases / stays the same

Key check: Did L change during the airtime? _____ Why or why not?

YOU DO Spinning Platform with Weights

A person stands on a frictionless turntable holding two 3.0 kg masses at arm's length (0.80 m from axis). The system spins at 2.0 rad/s. Total $I_{\text{person+turntable}}$ (without masses) = 4.0 kg·m².

(a) $I_{\text{total initial}}$ (include two masses at 0.80 m):

(b) Masses pulled to 0.20 m. $I_{\text{total final}}$:

(c) Find the new ω.

(d) By what factor did rotational KE change?

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EXIT TICKET & HOMEWORK

EXIT TICKET

A satellite has a 200 kg main body (model as a uniform sphere, $R = 0.50$ m) spinning at 2.0 rad/s about its own center of mass. It extends two solar panels (total mass 20 kg, modeled as point masses at 3.0 m from the center). What is the new spin rate?

HOMEWORK

1. Basic Conservation

A disk ($I = 0.40$ kg·m²) spinning at 8.0 rad/s is dropped onto an identical stationary disk (same I). They stick together. Find ω_f.

2. Neutron Star

When a massive star collapses, its core shrinks from $R \approx 700{,}000$ km to $R \approx 10$ km while approximately conserving angular momentum. If the original star rotated once every 30 days, estimate the rotation period of the neutron star. (Assume uniform spheres — $I = \frac{2}{5}MR^2$.)

3. Angular Impulse

A torque of 12 N·m is applied to a wheel ($I = 3.0$ kg·m²) for 4.0 s. The wheel starts from rest. (a) Find ΔL. (b) Find ω_f.

4. AP Comparison

Compare and contrast conservation of linear momentum ($p = mv$) and conservation of angular momentum ($L = I\omega$). When is each conserved? Can one be conserved while the other isn't? Give a specific example.

Next Class: Tomorrow is full rotation review. Bring everything — kinematics through angular momentum.

CALCULUS CORNER — THE ROTATIONAL FORM OF NEWTON'S SECOND LAW

Newton's second law has taken three forms this year: F = ma, F = dp/dt (Unit 4), and now **τ = dL/dt** with L = Iω. Zero net torque → dL/dt = 0 → L is conserved: that is WHY the skater speeds up when I drops. Tomorrow we differentiate L(t) directly and integrate angular impulse ∫τ dt = ΔL.