

ROTATING SYSTEMS DAY 1: ROTATIONAL KINETIC ENERGY

WARM-UP (3 MIN)

Flywheel Energy: A flywheel in a power plant spins at 3000 rpm and stores energy.

(a) Where is the energy stored? It's not moving across the room, so it has no translational KE. It's not elevated, so no U_g . It's not a spring, so no U_s . Where is it? _____

(b) Prediction: If you doubled the flywheel's spin rate, would the stored energy double? More than double? Less than double? _____

A NEW ENERGY ACCOUNT

ROTATIONAL KINETIC ENERGY

Equation:

$$K_{\text{rot}} = \frac{1}{2} I \omega^2$$

Where:

- I = rotational inertia
- ω = angular velocity
- K_{rot} = spinning energy

Compare: $K_{\text{trans}} = \frac{1}{2} m v^2$. Replace $m \rightarrow I$ and $v \rightarrow \omega$.

UPDATED ENERGY INVENTORY

$$E_{\text{mech}} = K_{\text{trans}} + K_{\text{rot}} + U_g + U_s$$

Type	Symbol	Formula	Stored Where
Translational KE	K_{trans}	$\frac{1}{2} m v^2$	Linear motion
Rotational KE	K_{rot}	$\frac{1}{2} I \omega^2$	Spinning motion
Gravitational PE	U_g	mgy	Height
Elastic PE	U_s	$\frac{1}{2} k x^2$	Spring

Physics in the Wild: Regenerative Braking

EV braking channels translational KE through rotating motor parts and into electrical storage. Flywheels use the same idea by storing energy as $\frac{1}{2} I \omega^2$.

WE DO Flywheel Energy

A solid steel cylinder (mass 50 kg, radius 0.40 m) spins at 3000 rpm.

(a) Find $I = \frac{1}{2} M R^2$.

(b) Convert 3000 rpm to rad/s.

(c) Find K_{rot} .

(d) If ω doubles, by what factor does K_{rot} change?

CALCULUS CORNER — WORK, ROTATED

Unit 3's work integral has a rotational twin: $\mathbf{W} = \int \boldsymbol{\tau} \, d\boldsymbol{\theta}$. Check one strip: $(\text{N}\cdot\text{m})(\text{rad}) = \text{J}$ — torque through a small angle deposits joules, exactly as $\mathbf{F}\cdot d\mathbf{x}$ did. For constant τ it collapses to $W = \tau \Delta\theta$, and the work-energy theorem lands on $\Delta K_{\text{rot}} = \Delta(\frac{1}{2} I \omega^2)$.

ROTATIONAL WORK

WORK DONE BY TORQUE

Constant-torque equation:

$$W = \tau \Delta\theta$$

Linear analog: $W = F\Delta x$.

Rotation work-energy theorem:

$$W_{\text{net}} = \Delta K_{\text{rot}} = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2$$

WE DO Spinning Up a Disk

A constant torque of 8.0 N·m is applied to a solid disk ($I = 0.60 \text{ kg}\cdot\text{m}^2$) initially at rest for 5.0 rev.

(a) Calculate $W = \tau\Delta\theta$.

(b) Use work-energy to find ω_f .

YOU DO Braking a Wheel

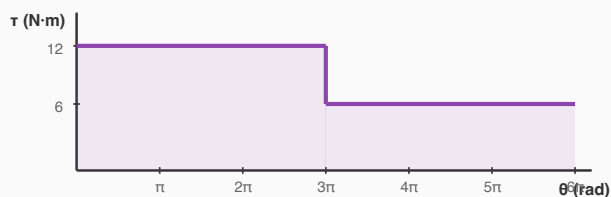
A wheel ($I = 1.2 \text{ kg}\cdot\text{m}^2$, $\omega_i = 30 \text{ rad/s}$) is brought to rest by a friction brake.

(a) How much work does the brake do?

(b) If the brake torque is 6.0 N·m, through how many radians does the wheel turn?

SKILL Graphical: Work from a τ vs θ Graph

The graph below shows the net torque on a wheel as it rotates through $6\pi \text{ rad}$. The wheel starts from rest with $I = 0.80 \text{ kg}\cdot\text{m}^2$.



(a) Work from $\theta = 0$ to $\theta = 3\pi$.

(b) Work from $\theta = 3\pi$ to $\theta = 6\pi$.

(c) Total work and ω at $\theta = 6\pi$.

ENERGY BAR CHARTS WITH ROTATION

Remember: From the Energy unit, you drew bar charts with K , U_g , and U_s . Now add a K_{rot} bar. The total stays constant (no friction) or decreases (with friction).

SKILL Bar Chart — Disk Drops on Axle

A solid disk (mass 2.0 kg, radius 0.10 m) drops 1.5 m while a string unwraps from its axle.

Draw the start and end energy bars:

Start	End
$K_{\text{trans}}, K_{\text{rot}}, U_g$	$K_{\text{trans}}, K_{\text{rot}}, U_g$

(a) Set up $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$, then use $v = \omega R$.

(b) Solve for v .

YOU DO Ball Rolling Down Ramp

A solid sphere (mass 3.0 kg, radius 0.08 m) rolls without slipping down a 2.0 m high ramp.

(a) Write the energy conservation equation.

(b) Use $I = \frac{2}{5}MR^2$ and $v = \omega R$ to find the bottom speed.

(c) Compare with a block sliding from the same height. Which is faster, and why?

EXIT TICKET & HOMEWORK

EXIT TICKET

A hoop ($I = MR^2$) and a disk ($I = \frac{1}{2}MR^2$) both have mass 1.0 kg and radius 0.20 m, and both spin at 10 rad/s.

(a) Find K_{rot} for each.

(b) Which stores more energy, and why?

HOMEWORK

1. Basic KE: A thin rod (mass 0.80 kg, length 1.2 m) rotates about one end at 5.0 rad/s. Use $I = \frac{1}{3}ML^2$.

2. Rotational Work: A torque of 15 N·m accelerates a wheel from rest to 120 rad/s. $I = 0.50 \text{ kg}\cdot\text{m}^2$.

(a) Find K_{rot} at 120 rad/s.

(b) Find the angle turned.

3. Rolling Energy Split: A solid cylinder rolls without slipping at $v = 4.0 \text{ m/s}$.

(a) What fraction of its total KE is translational?

(b) What fraction is rotational?

4. Looking Ahead: Does a bowling ball or a basketball reach the bottom of a ramp first, assuming both roll without slipping and have the same radius?

Tomorrow: Rolling Motion. We combine translational and rotational KE to solve the great ramp race.