ROTATION DAY 10: REVIEW

BIG IDEAS TO REMEMBER

- **The ladder, new letters:** $\omega = d\theta/dt$, $\alpha = d\omega/dt$; $\Delta\theta = \int \omega\, dt$. Every constant-α equation is Unit 1 with $x \to \theta$, $v \to \omega$, $a \to \alpha$.
- **Linear ↔ angular:** $v = \omega r$, $a_t = \alpha r$ — the rim moves faster than the hub at the same ω.
- **Torque:** $\tau = rF \sin\theta = \|r \times F\|$ — lever arm times force; sign from your CCW/CW convention (the right-hand rule in disguise).
- **Statics:** $\Sigma F = 0$ AND $\Sigma\tau = 0$ — about any pivot; pick the pivot that erases an unknown.
- **Inertia is an integral:** $I = \int r^2 dm$ — rod $ML^2/12$ (center) or $ML^2/3$ (end); hoop MR^2; disk $\frac{1}{2}MR^2$; and $I = I_{cm} + Md^2$ to move the axis.
- **Newton's 2nd, rotated:** $\Sigma\tau = I\alpha$.

A Method Choice

For each situation, name the tool (ladder / statics / $\tau = I\alpha$ / inertia integral) and one reason:

a. A sign hangs from a beam held by one cable — find the cable tension.

b. A flywheel's $\theta(t)$ is a polynomial — find α at $t = 2$ s.

c. A non-uniform baton spins about its light end — find its I.

d. A rope torque spins up a pulley from rest — find ω after 4 s.

B Ladder Check

A wheel's angle follows $\theta(t) = 2t^3 - 3t^2$ (radians, seconds). **(a)** Find $\omega(t)$ and $\alpha(t)$. **(b)** Evaluate both at $t = 2.0$ s. **(c)** At what time is the wheel momentarily at rest (after $t = 0$)?

MIXED PRACTICE

C Statics: Pick the Smart Pivot

A uniform 4.0 m plank (mass 20 kg) rests on two supports, one at the left end and one 1.0 m from the right end. A 40 kg toolbox sits at the CENTER of the plank. ($g = 9.8\ m/s^2$.)

(a) Which pivot choice kills the left support's force? **(b)** Find the force from the RIGHT support.

D Build the Inertia, Then Spin It

A uniform disk ($M = 8.0$ kg, $R = 0.50$ m) spins on a frictionless central axle. A 10 N force pulls tangentially on the rim. **(a)** Find I. **(b)** Find τ. **(c)** Find α.

E Two Axes, One Rod

A uniform rod has $M = 2.0$ kg and $L = 3.0$ m. **(a)** Find I about its center. **(b)** Find I about its end — by parallel axis OR by integral, your choice. **(c)** State the ratio without computing anything new.

CALCULUS CONNECTIONS

THE ROTATION UNIT'S TWO INTEGRALS

Integral	One strip is…	Units of one strip	Accumulates…
$\Delta\theta = \int \omega \, dt$	$\omega \cdot \Delta t$	(rad/s)(s) = rad	**angle turned**
$I = \int r^2 \, dm$	$r^2 \cdot dm$	$(m^2)(kg) = kg \cdot m^2$	**rotational inertia**

One runs over time, one over the body — but both are the same strip-sum limit you have used all year.

F Accumulate the Spin-Up

A turbine's angular velocity ramps as $\omega(t) = 4t$ (rad/s when t is in seconds) for 6.0 s. **(a)** Find the total angle turned, by integral — units of one strip first. **(b)** How many full revolutions is that?

G The Loaded Baton

A baton of length $L = 2.0$ m has $\lambda(x) = 3x$ (kg/m when x is in meters) and spins about its light end ($x = 0$). **(a)** Find its total mass. **(b)** Find $I = \int_0^L x^2 \lambda \, dx$. **(c)** Verify your numbers satisfy $I = ML^2/2$ — the loaded rod's signature.

AP-STYLE PRACTICE

H Short FRQ: Symbolic Reasoning

A uniform rod of mass M and length L is held horizontal, pivoted at one end, and released from rest. (Gravity acts at its center.)

a. **Derive** the angular acceleration at the instant of release, symbolically.

b. **Argue:** the answer contains no M. Why does the rod's mass not matter?

c. **Predict:** as the rod swings DOWN from horizontal, does α grow, shrink, or stay constant? Justify with the torque's lever arm.

I Reflection

Which of these can you least reliably do on your own — ladder problems, torque, statics, inertia integrals, or $\tau = I\alpha$? Name it, and write your plan to fix it before the unit test:

Test Reminder: Say which axis your I belongs to, show the lever arm on your diagram, and check units — $kg{\cdot}m^2$ for I, $N{\cdot}m$ for τ, rad/s^2 for α.