

NEWTON'S 2ND LAW FOR ROTATION

Warm-up (3 min): A grinding wheel ($I = 0.50 \text{ kg}\cdot\text{m}^2$) is spinning at 40 rad/s . A brake pad presses against the rim and exerts a friction force. The wheel stops in 8.0 s .

- (a) What is α ?
- (b) What is the net torque on the wheel?
- (c) If the wheel radius is 0.20 m , what friction force does the brake pad exert?

THE ROTATIONAL NEWTON'S SECOND LAW

THE COMPLETE PARALLEL

Linear Motion: $\Sigma F = ma \rightarrow$ Net force causes linear acceleration

Rotational Motion: $\Sigma \tau = I\alpha \rightarrow$ Net torque causes angular acceleration

$$\Sigma \tau = I\alpha$$

Where: $\Sigma \tau$ = net torque ($\text{N}\cdot\text{m}$), I = rotational inertia ($\text{kg}\cdot\text{m}^2$), α = angular acceleration (rad/s^2)

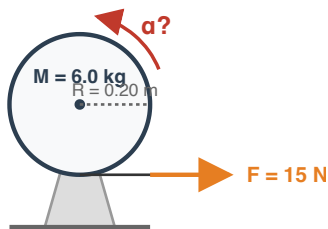
Key Insight: This is Newton's Second Law in disguise. Force $\rightarrow$ Torque. Mass $\rightarrow$ Rotational inertia. Acceleration $\rightarrow$ Angular acceleration. Same physics, different geometry.

PROBLEM-SOLVING: ROTATION WITH $\tau = I\alpha$

1. Identify the rotating object and its axis
2. Find I (from table or calculate using $I = \frac{1}{2}MR^2$ for a disk)
3. Identify all torques (with signs: CCW = +, CW = -)
4. Write $\Sigma \tau = I\alpha$
5. Solve for the unknown

EXERCISE 1: HAND PULLING A STRING ON A FLYWHEEL**WE DO Pure Rotation — No Hanging Mass**

A solid disk (mass 6.0 kg, radius 0.20 m) is mounted on a frictionless horizontal axle. A string is wrapped around the rim and pulled with a constant force of 15 N.



(a) Find I for the disk. ($I = \frac{1}{2}MR^2$)

$I = \underline{\hspace{2cm}} \text{ kg}\cdot\text{m}^2$

(b) The string pulls tangentially at the rim. Find the torque: $\tau = FR = \underline{\hspace{2cm}}$

(c) Apply $\Sigma\tau = I\alpha$ to find α .

$\alpha = \underline{\hspace{2cm}} \text{ rad/s}^2$

(d) Starting from rest, what is ω after 3.0 s?

(e) How many revolutions does the disk complete in those 3.0 s?

Notice: This was a one-object problem. The string force creates a torque, the disk has rotational inertia, and $\tau = I\alpha$ gives us the angular acceleration. No hanging mass, no coupled equations.

QUICK What If?

If the same 15 N force were applied at half the radius (wrapping the string around a smaller inner spool), what would happen to:

(a) The torque? (increase / decrease / same)

(b) The angular acceleration? (increase / decrease / same)

EXERCISE 2 (CONTINUED): THE "EFFECTIVE MASS" SHORTCUT**EFFECTIVE LINEAR MASS**

The useful idea is more general than this pulley problem. Any time a **linear force** pulls on a rotating object at radius R , start with:

$$\Sigma \tau = I\alpha \quad \longrightarrow \quad FR = I \left(\frac{a}{R} \right)$$

Now divide both sides by R :

$$F = \left(\frac{I}{R^2} \right) a$$

So whenever rotation is linked to linear motion through $a = \alpha R$, the rotational inertia shows up in the linear equation exactly like an added mass of $\frac{I}{R^2}$. That is the **Effective Linear Mass** (M_{eff}).

SYSTEM SHORTCUT Re-solving the Atwood Machine

Because the pulley's rotational inertia acts like an extra mass holding the string back, we can unroll the entire Atwood machine and treat it as a single linear system:

$$a = \frac{\Sigma F_{external}}{m_{system}} = \frac{m_{heavy}g - m_{light}g}{m_{heavy} + m_{light} + M_{eff}}$$

(a) What is the formula for the effective mass (I/R^2) of our solid disk pulley ($I = \frac{1}{2}MR^2$)?

(b) Calculate the numerical value of M_{eff} for our 4.0 kg disk.

$M_{eff} = \underline{\hspace{2cm}}$ kg

(c) Plug everything into the system equation above to find a . Does it match your answer from Page 3?

$a = \underline{\hspace{2cm}}$ m/s²

General rule: If a rotating object is being driven by a string, belt, or other tangential force and the motion is tied together by $a = \alpha R$, you can treat the rotational part as an extra linear mass of I/R^2 . In this Atwood machine, that means the 4.0 kg solid disk behaves like an extra 2.0 kg in the denominator.

EXERCISE 3: WINCH LIFTING A BUCKET**YOU DO Applied Torque + Translation**

A cylindrical winch (mass 8.0 kg, radius 0.12 m, $I = \frac{1}{2}MR^2$) is used to lift a 15 kg bucket. An applied torque of 25 N·m turns the crank.

(a) Write $\Sigma F = ma$ for the bucket (up = +): _____

(b) Write $\Sigma \tau = I\alpha$ for the winch: _____

(c) Use $a = \alpha R$ to solve for a :

HOMEWORK**1 Simple Application**

A torque of 12 N·m is applied to a wheel with $I = 0.80 \text{ kg}\cdot\text{m}^2$. (a) Find α . (b) Starting from rest, what is ω after 3.0 s? (c) How many revolutions in those 3.0 s?

2 Friction Brake

A flywheel ($I = 2.5 \text{ kg}\cdot\text{m}^2$, radius 0.30 m) spins at 120 rad/s. A brake applies a friction force of 40 N to the rim. (a) Find the braking torque. (b) Find α . (c) How long to stop?

3 Coupled System

A 1.0 kg mass hangs from a string wrapped around a solid disk (mass 3.0 kg, radius 0.08 m). Released from rest. (a) Find the acceleration. (b) Find the string tension. (Hint: $T \neq mg$!)

Tomorrow: Unit Review. Then Unit 6 extends the energy framework to spinning objects: $K_{\text{rot}} = \frac{1}{2}I\omega^2$.