

ROTATION DAY 7: ROTATIONAL INERTIA

Warm-Up (3 min): Prediction Task

A hoop and a solid disk have the same mass and radius. Both start from rest at the top of an identical ramp. Which reaches the bottom first?

(A) Hoop (B) Disk (C) Same time (D) Not enough info

Your prediction: _____

Reasoning: _____

WHAT RESISTS ROTATION?

THE PARALLEL

In linear motion: Force causes acceleration. $F = ma$. More mass $\rightarrow$ harder to accelerate.

In rotation: Torque causes angular acceleration. $\tau = I\alpha$. But what is I ?

Key insight: It's not just total mass — it's *where* the mass is located relative to the rotation axis.

ROTATIONAL INERTIA (MOMENT OF INERTIA)

$$I = \sum m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \dots$$

Where: m = mass of each piece (kg), r = distance from rotation axis (m)

Units: $\text{kg}\cdot\text{m}^2$

Key principle: Mass far from the axis contributes *much more* to I than the same mass near the axis (because of the r^2 dependence!).

WE DO Point Masses on a Rod

A lightweight rod has three masses attached: 2.0 kg at 0.10 m, 1.0 kg at 0.20 m, and 3.0 kg at 0.30 m from the rotation axis.



(a) Calculate I for the system.

$I =$ _____ $\text{kg}\cdot\text{m}^2$

(b) If the 3.0 kg mass moves to 0.10 m, what is the new I ?

$I_{\text{new}} =$ _____ $\text{kg}\cdot\text{m}^2$

(c) Which arrangement is easier to spin up? Why?

COMMON SHAPES — THE REFERENCE TABLE

ROTATIONAL INERTIA OF STANDARD SHAPES

Shape	Axis	I
Point mass	Distance r from axis	mr^2
Thin hoop/ring	Through center	MR^2
Solid disk/cylinder	Through center	$\frac{1}{2}MR^2$
Solid sphere	Through center	$\frac{2}{5}MR^2$
Thin rod	Through center	$\frac{1}{12}ML^2$
Thin rod	Through end	$\frac{1}{3}ML^2$

Note: You don't need to memorize these — they'll be given on the AP exam. But understand the *pattern*: mass spread farther from the axis → larger I. That's why $I_{\text{hoop}} > I_{\text{disk}} > I_{\text{sphere}}$ (all same M and R).

Physics in the Wild: Biomechanics

Tightrope walker: Why carry a 10-meter pole? It puts mass *far* from the pivot (your feet), creating a huge I . Since $\alpha = \tau/I$, large I means you tip *slowly*, giving time to correct.

Sprinter: Why bend your knees when running? Bending brings mass *closer* to the hip joint, *decreasing* I , so your legs rotate faster for the same muscle torque.

SKILL Ranking Task

Rank these objects from **largest to smallest** rotational inertia. All have mass M and relevant dimension R or L.

(A) Solid disk about center (B) Hoop about center (C) Solid sphere about center (D) Point mass at distance R

Ranking: _____

WE DO Revisit the Warm-Up: Hoop vs. Disk Race

Both hoop and disk have mass M and radius R. Hoop: $I = MR^2$. Disk: $I = \frac{1}{2}MR^2$.

(a) Which has larger rotational inertia?

(b) Which resists angular acceleration more?

(c) On a ramp, gravitational PE converts to both translational KE ($\frac{1}{2}mv^2$) and rotational KE ($\frac{1}{2}I\omega^2$). The object with larger I "wastes" more energy on spinning. Which object has more translational speed at the bottom?

(d) Does the race result depend on mass? On radius? Explain.

QUALITATIVE REASONING: WHERE THE MASS SITS MATTERS

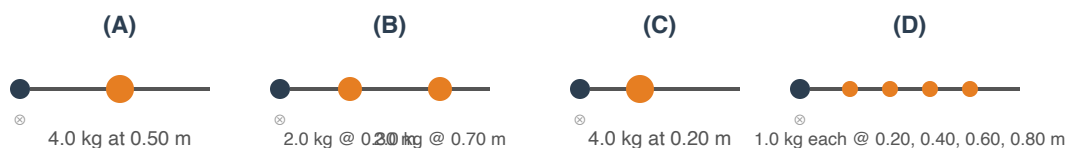
THE HAMMER GRIP TEST

Hold a hammer by the handle and waggle it back and forth. Now flip it and hold it by the head. **Which grip makes it easier to rotate?**

The total mass is the same either way — but the *distribution* of mass relative to your wrist (the axis) changes dramatically. When the heavy head is far away, I is large. When the heavy head is near the axis, I is small.

SKILL Ranking Task — Mass Distribution

All systems below have total mass $M = 4.0$ kg and rotate about the axis shown ($\otimes$). Rank from **largest to smallest** rotational inertia. Justify each choice using the r^2 reasoning.



Ranking (largest $I \rightarrow$ smallest I): _____

Justification:

WE DO Proportional Reasoning — How Does Doubling r Affect I ?

A point mass m is at distance r from the axis: $I = mr^2$.

(a) If r doubles (mass stays the same), by what factor does I change?

(b) If m doubles (distance stays the same), by what factor does I change?

(c) A 2.0 kg mass sits at 0.30 m from the axis. You move it to 0.90 m. By what factor did I change? Calculate both values to verify.

Tomorrow: where today's I values COME FROM — the integral $I = \int r^2 dm$. Then Day 9 combines τ and I into Newton's Second Law for Rotation ($\tau = I\alpha$).

EXIT TICKET & HOMEWORK

EXIT TICKET

Three objects roll down a ramp: a solid sphere, a solid cylinder (disk), and a hoop. All have the same mass and radius.

(a) Rank them by finish time (fastest to slowest):

(b) In one sentence, explain why the ranking doesn't depend on mass or radius.

HOMEWORK

1 Point Masses in a Square

Four 0.50 kg masses are at the corners of a square (side length 0.40 m). Find I about an axis through the center of the square, perpendicular to the square. (Hint: what is r for each mass?)

$I = \underline{\hspace{2cm}} \text{ kg}\cdot\text{m}^2$

2 Axis Location Matters

A thin uniform rod (mass 2.0 kg, length 1.0 m) can rotate about (A) an axis through the center ($I = \frac{1}{12}ML^2$), or (B) an axis through one end ($I = \frac{1}{3}ML^2$).

(a) Calculate I for each axis:

$I_{\text{center}} = \underline{\hspace{2cm}} \text{ kg}\cdot\text{m}^2$, $I_{\text{end}} = \underline{\hspace{2cm}} \text{ kg}\cdot\text{m}^2$

(b) Why is I_{end} four times larger? Explain using mass distribution — which axis puts more mass far away?

3 Conceptual: Baseball Bat

You hold a baseball bat (a) by the handle (normal grip), (b) by the barrel (choked up). Which grip makes the bat harder to swing (rotate) quickly? Explain using rotational inertia.