

ROTATION DAY 3: TORQUE & LEVER ARM

WARM-UP (3 min): You need to loosen a stuck bolt. Rank these strategies by effectiveness (easiest to hardest):

- **A:** Push with 50 N using a long wrench, at the end, perpendicular to the wrench.
- **B:** Push with 100 N using a short wrench, at the end, perpendicular.
- **C:** Push with 100 N using a long wrench, but pushing at 30° to the wrench.
- **D:** Push with 100 N at the middle of the long wrench, perpendicular.

Prediction: ____ > ____ > ____ > ____ After we learn the equation, we'll check your ranking.

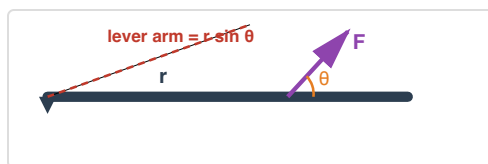
WHAT MAKES THINGS ROTATE?

TORQUE

$$\tau = rF \sin \theta$$

Where:

- r = distance from pivot to where force is applied (m)
- F = magnitude of force (N)
- θ = angle between the position vector and force vector
- τ = torque (N·m — NOT joules!)



Two equivalent ways to think about it:

(1) $\tau = (r \sin \theta)F = (\text{lever arm}) \times (\text{full force})$, or

(2) $\tau = r(F \sin \theta) = (\text{full distance}) \times (\text{perpendicular component of force})$.

Both give the same answer.

WE DO Check the Warm-Up Rankings

Now calculate τ for each bolt scenario:

Strategy	r (m)	F (N)	θ	$\tau = rF \sin \theta$ (N·m)
A	0.30	50	90°	
B	0.15	100	90°	
C	0.30	100	30°	
D	0.15	100	90°	

Final ranking: ____ > ____ > ____ > ____ Did your prediction match?

CALCULUS CORNER — TORQUE IS A CROSS PRODUCT (READ-ONLY)

$\tau = rF \sin \theta$ is the magnitude of a vector operation: $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$, the cross product of the position vector (axis to the point where the force lands) with the force. $\|\mathbf{r} \times \mathbf{F}\| = rF \sin \theta$ — the $\sin \theta$ is built in, and the right-hand rule assigns the sign your CCW/CW convention has been tracking by hand. Same physics, tighter packaging; you'll compute cross products directly later in the course.

SIGN CONVENTION FOR TORQUE**POSITIVE AND NEGATIVE TORQUE**

- **CCW = positive τ** (like standard math angles)
- **CW = negative τ**



Common mistake: Students assign the sign based on the direction of the force (up = positive). Wrong! The sign depends on which WAY the force would make the object rotate around the pivot.

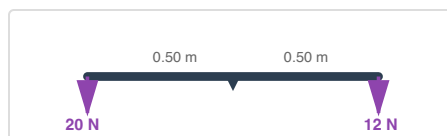
Physics in the Wild: The Cheater Bar

When a mechanic can't break a bolt loose, they slide a long pipe over the wrench handle — called a "cheater bar." It doubles or triples r while F stays the same, so torque skyrockets. Some cheater bars are over a meter long. Medieval castle doors had long handles for exactly the same reason: more torque from a weaker push.

Quick math: If a mechanic applies 200 N with a 0.30 m wrench and can't break a bolt free, what happens when they add a 0.60 m pipe? How much torque now?

WE DO Beam with Two Forces

A 1.0 m beam is pivoted at its center. A 20 N force pushes DOWN at the left end; a 12 N force pushes DOWN at the right end.



(a) Torque from left force (with sign):

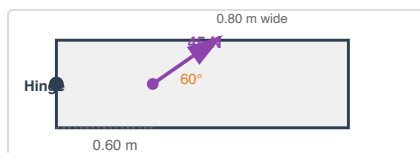
(b) Torque from right force (with sign):

(c) Net torque:

(d) Which way will the beam rotate?

YOU DO Angled Force on a Door

A door is 0.80 m wide (hinged on left). You push with 45 N at a 60° angle to the door surface, at a point 0.60 m from the hinges.

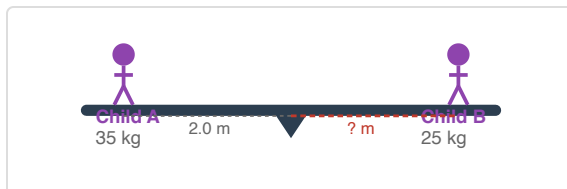


(a) Draw the lever arm on the diagram above. (Use a dashed line.)

(b) Calculate the torque about the hinge.

MULTIPLE TORQUES — NET TORQUE**WE DO Playground Seesaw**

Two children sit on a seesaw (pivot in middle). Child A (mass 35 kg) sits 2.0 m from the pivot. Child B (mass 25 kg) sits on the other side.



(a) Find the torque from Child A's weight about the pivot.

(b) How far from the pivot must Child B sit to balance the seesaw? (Set $\Sigma\tau = 0$.)

YOU DO Three Forces

A horizontal beam is pivoted at the left end. Forces: 30 N down at 0.20 m from pivot, 50 N up at 0.60 m from pivot, 20 N down at 1.0 m from pivot.



(a) Calculate each torque (with sign):

(b) Find net torque:

(c) Which way does the beam tend to rotate?

This is the foundation for statics (Day 4). If an object isn't rotating, the net torque **MUST** be zero. Tomorrow we use this to solve for unknown forces.

EXIT TICKET & HOMEWORK

EXIT TICKET: A wrench is 0.25 m long. You apply 80 N at the end, perpendicular to the wrench.

(a) Find the torque:

(b) If you instead push at 45° to the wrench, what torque do you produce?

HOMEWORK

PROBLEM 1 Conceptual — Revolving Door

A force is applied to a revolving door. In which case is the torque greatest?

- (a) Pushing at the outer edge, perpendicular.
- (b) Pushing at the outer edge, at 45° .
- (c) Pushing halfway to center, perpendicular.
- (d) Pushing at the outer edge, parallel to the door.

Circle answer: _____ Explain in one sentence:

PROBLEM 2 Lever Arm Practice

A mechanic uses a 0.40 m wrench and applies 120 N at 75° to the wrench. Calculate the torque.

PROBLEM 3 Two-Force Balance

A uniform meter stick (mass 0.15 kg) is supported at the 40 cm mark. A 0.50 kg mass hangs at the 10 cm mark. Where should a 0.30 kg mass be placed to balance the stick? (Remember: the stick's weight acts at its center of mass — the 50 cm mark.)

PROBLEM 4 Looking Ahead

Tomorrow we tackle statics — objects that don't translate AND don't rotate. What TWO conditions must both be true? (Hint: one involves forces, one involves torques.)

Tomorrow: Static Equilibrium. Two conditions: $\Sigma F = 0$ AND $\Sigma \tau = 0$. It's Newton's Laws meets torque.