ROTATION DAY 1: ANGULAR KINEMATICS

Warm-up (3 min): A clock's minute hand (15 cm) completes one revolution in 60 min. A point halfway along the hand is at 7.5 cm.

(a) How far does the tip travel in one full revolution?

(c) Which point is moving faster in m/s?

(b) How far does the halfway point travel?

(d) What quantity is the SAME for both?

ANGLES IN RADIANS

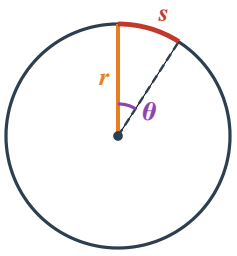
THE RADIAN

Instead of degrees, we measure rotation in **radians**:

$$\theta\ (\text{rad}) = \frac{s}{r}$$

where: s = arc length (m), r = radius (m), θ = angle (rad)

Full revolution = 2π rad = 360° | deg × $\frac{\pi}{180}$ = rad



θ = s / r (1 rad shown)

WE DO Convert These Angles

(a) Convert 45° to radians

(b) Convert $3\pi/2$ rad to degrees

(c) How many radians in 2.5 revolutions?

CALCULUS CORNER — THE SAME LADDER, NEW LETTERS

Unit 1's derivative ladder runs unchanged here: **ω = dθ/dt** and **α = dω/dt** — slopes of the θ-t and ω-t graphs — and the accumulations run back up: Δθ = ∫ω dt (units check: (rad/s)(s) = rad). Every constant-α equation on this page is the Unit 1 derivation with x→θ, v→ω, a→α. Given θ(t) as a polynomial, differentiate for ω(t) and α(t) exactly as you did for x(t).

ANGULAR VELOCITY & ACCELERATION

THE PARALLEL

Linear motion and rotation have identical structure. Just swap variables:

Quantity	Linear Symbol	Quantity	Angular Symbol
Position	x	Angular position	θ
Velocity	$v = \frac{\Delta x}{\Delta t}$	Angular velocity	$\omega = \frac{\Delta \theta}{\Delta t}$
Acceleration	$a = \frac{\Delta v}{\Delta t}$	Angular accel.	$\alpha = \frac{\Delta \omega}{\Delta t}$
Units	m, m/s, m/s²	Units	rad, rad/s, rad/s²

Key insight: Every kinematic equation you memorized has a rotational twin. Just swap $x \to \theta$, $v \to \omega$, $a \to \alpha$.

Physics in the Wild: Your Car's Tachometer

That gauge on the dashboard labeled "RPM × 1000"? It reads angular velocity — your engine's ω in real time. When you floor the gas in first gear, the needle sweeps from ~1000 RPM to ~5000 RPM in about 6 seconds. That sweep IS angular acceleration.

THE FOUR KINEMATIC EQUATIONS

ROTATIONAL FORM (CONSTANT ANGULAR ACCELERATION)

$$\omega = \omega_0 + \alpha t$$

$$\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

$$\theta = \tfrac{1}{2}(\omega_0 + \omega)t$$

WE DO Spinning Up a Wheel

A bicycle wheel starts from rest and accelerates uniformly at $\alpha = 4.0\ \text{rad/s}^2$.

(a) What is ω after 3.0 s?

(b) How many radians does it turn?

(c) How many full revolutions is that? (Hint: $1\ \text{rev} = 2\pi\ \text{rad}$)

YOU DO Braking Turntable

A turntable spins at 33⅓ rpm (common vinyl speed). When turned off, it decelerates uniformly and stops in 8.0 s.

(a) Convert 33⅓ rpm to rad/s

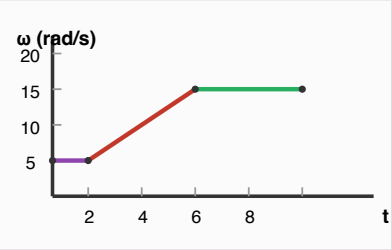
(b) Find α

(c) How many revolutions does it make while stopping?

READING ANGULAR VELOCITY GRAPHS

QUICK Interpreting an ω-t Graph

Study this angular velocity vs. time graph:



(a) What is α during 0–2 s?

(b) What is α during 2–5 s?

(c) What is the total angular displacement from 0 to 8 s? (Hint: area under the ω-t graph.)

Graph insight: Just like v-t graphs: slope = acceleration, area = displacement. Replace v with ω and x with θ.

EXIT TICKET & HOMEWORK

EXIT Fan Blade Acceleration

A fan blade accelerates from $\omega_0 = 2.0\ \text{rad/s}$ to $\omega = 10.0\ \text{rad/s}$ in $t = 4.0\ \text{s}$. Find α and the number of revolutions.

HOMEWORK PROBLEMS

1 Unit Conversion

(a) Convert 120 rpm to rad/s

(b) Convert 15 rad/s to rpm

(c) A car tire (radius 0.32 m) rotates at 80 rad/s. What is the car's speed in m/s? (Preview: tomorrow we connect linear and angular motion.)

2 Centrifuge Spin-up

A centrifuge starts from rest and reaches 12,000 rpm in 30 s with constant α.

(a) Find α in rad/s²

(b) How many revolutions during spin-up?