

OSCILLATIONS DAY 5: ENERGY OF SHM

Physics in the Wild: Taipei 101's Secret Weapon

Inside the world's tallest skyscraper (at the time) hangs a 730-ton steel ball — a tuned mass damper. During typhoons, the building sways and the pendulum swings in opposition, absorbing the building's kinetic energy as its own potential energy. Energy sloshes back and forth between the building and the damper, keeping the structure safe.

Emergence: Energy transfers back and forth, $KE \rightarrow PE \rightarrow KE$, yet the total stays constant. The individual energies change continuously while the total stays fixed (in the ideal, undamped model). This conservation is what makes SHM useful — from dampers to clocks to music.

ENERGY IN SHM

- $E_{\text{total}} = \frac{1}{2}kA^2$ (set once by initial conditions — never changes)
- $PE = \frac{1}{2}kx^2$ (elastic potential energy)
- $KE = \frac{1}{2}mv^2$
- $KE + PE = E_{\text{total}}$ at all times
- $v_{\text{max}} = A\sqrt{k/m} = A\omega$ (occurs at $x = 0$)

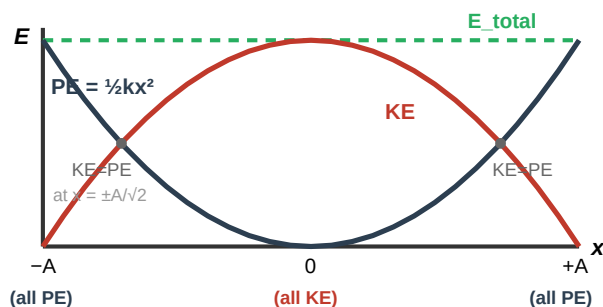
WE DO

Exercise 1: A 0.50 kg mass on a spring ($k = 200 \text{ N/m}$) oscillates with amplitude 0.10 m. Find: (a) E_{total} (b) v_{max} (c) v when $x = 0.05 \text{ m}$

CALCULUS CORNER — THE LANDSCAPE YOU ALREADY OWN

Yesterday's energy landscape, revisited: $U(x) = \frac{1}{2}kx^2$ is the parabolic valley from Unit 3 Day 7, and the total $E = \frac{1}{2}kA^2$ is the dashed line touching it at $x = \pm A$ — the turning points. The K and U bars trade all cycle while their sum never moves, and $v = \omega\sqrt{A^2 - x^2}$ is just $K = E - U$ solved for speed.

ENERGY VS POSITION



KEY POSITIONS

Position	KE	PE	v	a
$x = 0$	E_{total}	0	v_{max}	0
$x = \pm A/2$	$\frac{3}{4}E$	$\frac{1}{4}E$	$(\sqrt{3}/2)v_{\text{max}}$	$\frac{1}{2}a_{\text{max}}$
$x = \pm A$	0	E_{total}	0	a_{max}

WE DO Exercise 2: At $x = A/2$, what fraction of the total energy is KE? What fraction is PE?

YOU DO Exercise 3: A spring system has $E_{\text{total}} = 4.0 \text{ J}$, $k = 200 \text{ N/m}$, $m = 0.50 \text{ kg}$. Find: (a) A
 (b) v at $x = 0.10 \text{ m}$ (c) The position where $KE = PE$

ENERGY IN A PENDULUM

PENDULUM ENERGY

- At max angle: all PE ($h = L - L \cos \theta$), no KE $\rightarrow mgh = mgL(1 - \cos \theta)$
- At bottom: all KE, no PE $\rightarrow \frac{1}{2}mv_{\max}^2 = mgL(1 - \cos \theta)$
- $v_{\max} = \sqrt{2gL(1 - \cos \theta)}$

SKILL Exercise 4: Pendulum Drop — A 0.30 kg pendulum of length 1.2 m is released from $\theta = 25^\circ$. Find: (a) The height h above the lowest point (b) $v_{\max}$ at the bottom (c) The period T

ENERGY BAR DIAGRAMS

Drawing Energy Bars: Draw KE and PE bars at different positions. They must always stack to the same total height. This visual tool works for SHM just like it did in the Energy unit.

YOU DO Exercise 5: Energy Bars — Draw five sets of KE/PE energy bars for a mass on a spring at positions: $x = -A$, $x = -A/2$, $x = 0$, $x = +A/2$, $x = +A$.

-A

-A/2

0

+A/2

+A

Each bar represents total energy; divide into KE (red) and PE (blue).

EXIT TICKET & HOMEWORK

EXIT Spring Launcher — A spring ($k = 800 \text{ N/m}$) is compressed 0.05 m and launches a 0.020 kg ball vertically. (a) Find the elastic PE stored. (b) Find the speed of the ball as it leaves the spring. (c) Find the maximum height reached. (Assume all PE $\rightarrow$ KE $\rightarrow$ gravitational PE)

HOMEWORK PROBLEMS

[1] Mass-Spring System — A mass-spring system has $v_{\text{max}} = 1.5 \text{ m/s}$, $m = 0.40 \text{ kg}$, $k = 100 \text{ N/m}$. Find: (a) A (b) E_{total} (c) v at $x = A/2$

[2] Pendulum Comparison — A pendulum ($L = 2.0 \text{ m}$) swings through an angle of 10° . Find v_{max} at the bottom. Compare this to a pendulum with $L = 2.0 \text{ m}$ swinging through 20° .

[3] Half-Speed Position — At what displacement is the speed of an oscillator half of its maximum speed? Express as a fraction of A . (Hint: use energy conservation.)

Tomorrow: Lab Day! You'll measure periods of springs and pendulums, then use linearization to extract k and g from your data. Bring a calculator and graph paper.