OSCILLATIONS DAY 4: GRAPHS & THE SMALL ANGLE APPROXIMATION

Physics in the Wild: Reading Oscillations
Doctors read EKGs (heart oscillations). Seismologists read seismograms (earth oscillations). Audio engineers read waveforms (sound oscillations). All of them are reading SHM graphs.

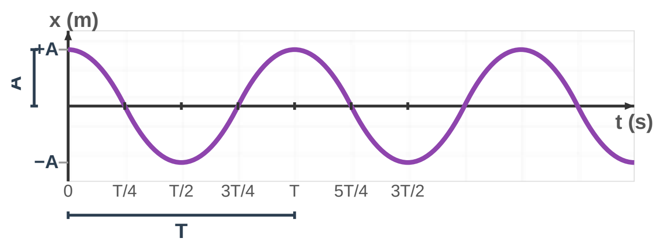
Emergence thread: *Universality — the same sinusoidal pattern appears in springs, pendulums, sound, light, molecular vibrations, and electrical circuits. When many different systems produce the exact same pattern, that pattern is emergent — it belongs to the math, not to any one physical system.*

POSITION VS TIME

$$x(t) = A\cos\left(\frac{2\pi t}{T}\right)$$

A = amplitude (max displacement), **T** = period, **f** = 1/T

If released from maximum positive displacement, the object starts at x = +A when t = 0.



WE DO **Exercise 1: Reading the Graph**

From the graph above, identify: (a) The amplitude A (b) The period T (c) The frequency f (d) At what times is the object at x = 0?

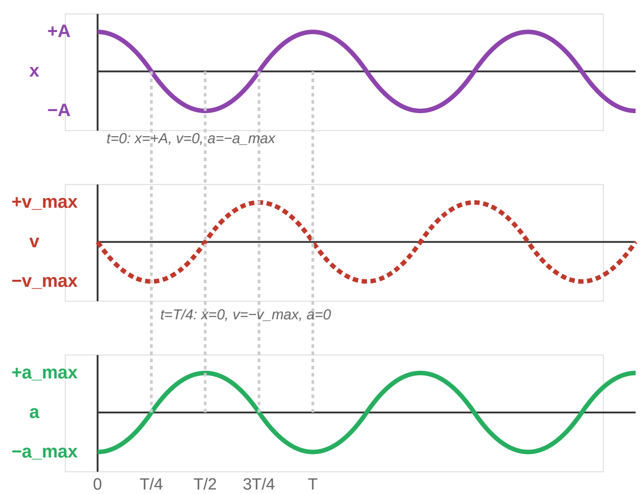
PHASE RELATIONSHIPS: X, V, AND A

THE KEY RELATIONSHIPS

$$v_{\max} = \frac{2\pi A}{T} = A\omega \quad \text{where} \quad \omega = \frac{2\pi}{T}$$

$$a_{\max} = \left(\frac{2\pi}{T}\right)^2 A = A\omega^2$$

- **v is maximum** when x = 0 (passing through the center)
- **v is zero** when x = ±A (momentarily stopping at turning points)
- **a is maximum** when x = ±A (maximum stretch = maximum restoring force)
- **a is zero** when x = 0 (at equilibrium, no restoring force)



Key insight: Notice that the peaks of the velocity graph happen right as position crosses zero. They are **OPPOSITE** — not in sync! At the endpoints, the object stops momentarily (v = 0) before reversing. At the center, it is moving fastest.

UNIVERSALITY & THE SMALL ANGLE ASSUMPTION

A system undergoes simple harmonic motion when its net force is a linear restoring force, $F = -kx$, over the range of motion. A mass on a spring satisfies this exactly in our ideal model; a pendulum satisfies it only for small angles. Let's check whether a pendulum obeys this condition.

THE SMALL ANGLE TRICK

For a pendulum, gravity pulls down, but the string forces it to swing in an arc. The component of gravity pulling it *back to the center* is:

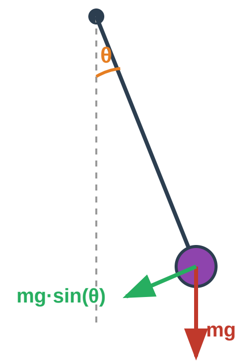
$$F_{\text{restore}} = -mg\sin(\theta)$$

Because of the $\sin(\theta)$, the force is **not** perfectly proportional to displacement. A pendulum is not perfectly SHM!

BUT... if θ is small (less than 15°), $\sin(\theta) \approx \theta$ (when measured in radians). Since arc length $x = L\theta$, we can substitute $\theta = x/L$:

$$F_{\text{restore}} \approx -mg\left(\frac{x}{L}\right) = -\left(\frac{mg}{L}\right)x$$

This matches Hooke's Law ($F = -kx$) perfectly! The pendulum's **"effective spring constant" is** $k = mg/L$.



The Math Trick: If we substitute the effective spring constant $k = \frac{mg}{L}$ into the spring period formula $T = 2\pi\sqrt{\frac{m}{k}}$, we get $T = 2\pi\sqrt{\frac{m}{mg/L}} = 2\pi\sqrt{\frac{L}{g}}$. Notice the mass cancels out entirely!

CONCEPT **Exercise 2: Proving the Approximation**

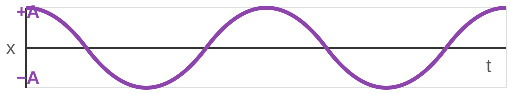
Let's test it. If $\theta = 10^\circ$:
(a) Convert 10° to radians (multiply by $\pi/180$).
(b) Calculate $\sin(10^\circ)$ in degree mode.
Are they close? What if you try it with 45°?

YOU DO **Exercise 3: Effective Spring Constant**

A 0.50 kg pendulum bob hangs from a 1.2 m string. What is its "effective spring constant"? What is its period of oscillation?

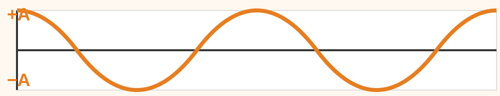
WE DO **Exercise 4: From x-t to v-t**

Given the x-t graph below (which starts at +A), sketch the corresponding v-t graph.



EXIT **Graph Interpretation**

The graph shows x-t for an oscillator with A = 0.04 m and T = 0.8 s.



(a) Find f and ω. (b) Find $v_{\max}$ and $a_{\max}$. (c) At t = 0.2 s ($T/4$), what are the values of x, v, and a?

HOMEWORK PROBLEMS

HW 1 **Graph and Equations**

A graph shows x(t) for SHM with peaks at x = 0.06 m and a period of T = 1.0 s.

(a) Find $v_{\max}$.
(b) Find $a_{\max}$.

HW 2 **Reverse Engineering**

An object in SHM has $v_{\max} = 0.8\ \mathrm{m/s}$ and $a_{\max} = 5.0\ \mathrm{m/s}^2$. Find the amplitude A and period T.

Hint: $v_{\max} = A\omega$, $a_{\max} = A\omega^2$, so what is $a_{\max}/v_{\max}$?

Tomorrow: Energy! We'll connect Kinetic and Potential Energy to these graphs and see that energy conservation explains exactly *why* velocity is maximum at x = 0.