OSCILLATIONS DAY 3: PENDULUMS

Physics in the Wild: From Galileo to Grandfather Clocks

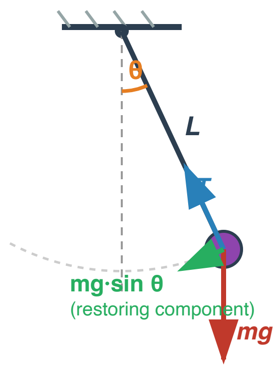
Legend says Galileo timed the swinging chandelier in the Pisa cathedral using his pulse. He noticed something remarkable: no matter how wide the swing, each oscillation took the same time. This observation led to the pendulum clock — the most accurate timekeeper for 300 years.

Emergence thread: *"The pendulum's period is independent of mass. From F = mg (which depends on mass) emerges T = 2π√(L/g) (which doesn't). The mass cancels when you work through the physics. The whole behaves differently than you'd predict from the parts."*

PERIOD OF A PENDULUM

$$T = 2\pi\sqrt{\frac{L}{g}}$$

- L = length (m), g = gravitational acceleration (m/s²)
- Valid for small angles (θ < ~15°)
- **Mass does NOT appear!** Heavy and light pendulums of the same length swing at the same rate.



WE DO **Exercise 1: A pendulum is 0.80 m long. Find its period on Earth ($g = 9.8\ \mathrm{m/s^2}$).**

YOU DO **Exercise 2: A grandfather clock needs a period of exactly 2.00 s. How long should the pendulum be?**

OSCILLATIONS DAY 3: PENDULUMS (CONT.)

QUICK **Exercise 3: Predict WITHOUT calculating: What happens to the period if you...**

- **(a) double the length?**
- **(b) quadruple the length?**
- **(c) take it to the Moon where $g = 1.62\ \mathrm{m/s^2}$?**
- **(d) double the mass of the bob?**

WE DO **Exercise 4 (Pendulum on Mars): A 1.5 m pendulum swings on Mars where $g = 3.71\ \mathrm{m/s^2}$. Find T. Compare to the same pendulum on Earth.**

YOU DO **Exercise 5 (Mystery Planet): A 0.60 m pendulum has a period of 2.1 s on an unknown planet. Find g for this planet. Which planet might this be?**

Geophysical Applications: This is how geophysicists measure local gravity variations — swing a pendulum, time it carefully, solve for g. Oil deposits, underground caverns, and ore bodies all create tiny g variations that pendulums can detect.

OSCILLATIONS DAY 3: PENDULUMS (CONT.)

COMPARING TWO OSCILLATORS

SPRINGS VS PENDULUMS — SIDE BY SIDE

Property	Mass-Spring	Pendulum
Restoring force	$F = -kx$	$F \approx -\frac{mg}{L}x$
Period	$T = 2\pi\sqrt{\frac{m}{k}}$	$T = 2\pi\sqrt{\frac{L}{g}}$
T depends on	mass, spring constant	length, gravity
T independent of	amplitude	amplitude, mass
"Effective k"	k	$\frac{mg}{L}$

Universal SHM: Both systems are SHM because both have a restoring force proportional to displacement. The math is identical — only the symbols change. This is universality in action.

SKILL **Exercise 6 (Same Period Challenge): A 0.40 kg mass on a spring ($k = 160\ \mathrm{N/m}$) oscillates with the same period as a pendulum. How long is the pendulum? (Work on Earth.)**

YOU DO **Exercise 7 (Which is faster?): System A: 0.30 kg on a spring with $k = 120\ \mathrm{N/m}$. System B: Pendulum with $L = 0.25\ \mathrm{m}$. Which completes one oscillation first? By how much time?**

EXIT TICKET & HOMEWORK

EXIT **Grandfather Clock Repair: A grandfather clock's pendulum ticks with $T = 2.00\ \mathrm{s}$ in New York. The clock is shipped to Denver (higher altitude, g slightly smaller).**

- (a) Does the clock run fast or slow? Explain.
- (b) Should the pendulum be made longer or shorter to correct it?
- (c) If $g_{\mathrm{Denver}} = 9.796\ \mathrm{m/s}^2$ and $g_{\mathrm{NY}} = 9.803\ \mathrm{m/s}^2$, find the new period in Denver (use the original L).

HOMEWORK PROBLEMS

[1] Find the period of a 2.0 m pendulum on (a) Earth, (b) Jupiter ($g = 24.8\ \mathrm{m/s}^2$), (c) the ISS ($g \approx 0$). *What happens when $g \to 0$?*

[2] A child on a swing ($L = 3.0\ \mathrm{m}$) completes 10 swings in 35 s. (a) Find T. (b) Compare to the theoretical value. (c) Why might they differ?

[3] A spring has $k = 50\ \mathrm{N/m}$. What mass gives the same period as a 1.0 m pendulum?

Tomorrow: Graphs! We'll see how x, v, and a change during oscillation. The phase relationships are tricky — velocity is NOT maximum when displacement is maximum. It's the opposite!

CALCULUS CORNER — THE SMALL-ANGLE TRADE-OFF (READ-ONLY)

Where does $T = 2\pi\sqrt{L/g}$ come from? Torque about the pivot: $\tau = -mgL\sin\theta$, and for small angles $\sin\theta \approx \theta$ (within 1% below ~14°). Then $I\alpha = -mgL\theta$ with $I = mL^2$ gives $\alpha = -(g/L)\theta$ — yesterday's differential equation with g/L in place of k/m, so $\omega = \sqrt{g/L}$. The formula is only as good as the approximation: big swings genuinely run slow.