OSCILLATIONS DAY 1: SPRINGS & EMERGENT PHENOMENA

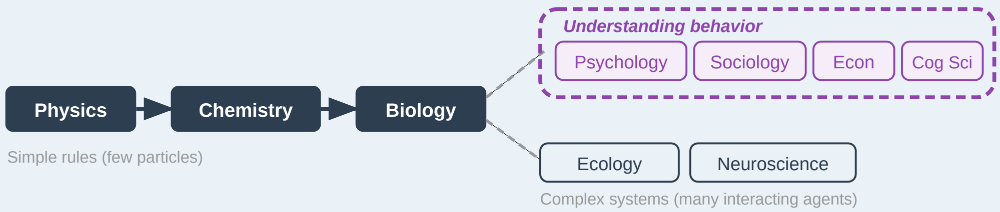
From simple rules to complex behavior

The Algodoo Experiment

Drop one bouncy ball in *Algodoo*—you can track it with kinematics. Drop **1,000** and a new pattern emerges: the particles act like a **gas**, creating collective *pressure* and **temperature**. No single ball has "temperature"—that concept only exists for the group.

WHAT IS EMERGENCE?

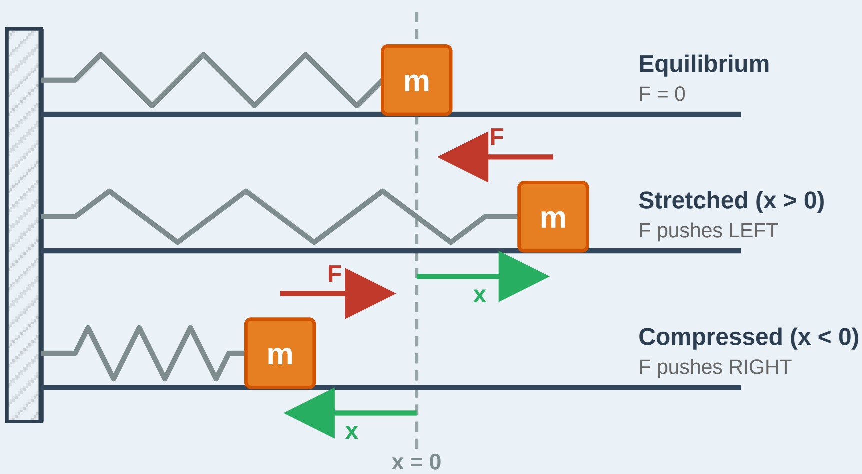
An **emergent phenomenon** is a complex behavior that arises from many simple interactions—something the individual parts alone could never produce. Emergence is a useful lens across science — simple rules can produce a stable, larger-scale pattern:



HOOKE'S LAW

$$F_s = -kx$$

k = spring constant (N/m), x = displacement from equilibrium (m). The negative sign means it's a **restoring force**: it always pulls the mass *back* toward equilibrium.



WE DO **Exercise 1: Restoring Force**

A spring with $k = 250\ \mathrm{N/m}$ is stretched $0.08\ \mathrm{m}$ from equilibrium. Find the magnitude of the restoring force.

HOOKE'S LAW & ANGULAR FREQUENCY

The minus sign sets the **direction**. When you calculate **magnitude**, use $|F| = k|x|$. Push right → force left. Push left → force right.

YOU DO **Exercise 2: Find the Spring Constant**

A spring exerts a force of $45\ \mathrm{N}$ when compressed $0.15\ \mathrm{m}$. Find the spring constant k.

ANGULAR FREQUENCY (Ω)

$$\omega = \sqrt{\frac{k}{m}}$$

ω (omega) tells us how fast the system cycles through its oscillation, measured in **radians per second (rad/s)**.

- Bigger k (stiffer spring) → larger ω (faster oscillation)
- Bigger m (heavier mass) → smaller ω (slower oscillation)

CONCEPT **Exercise 3: Proportionality**

If you wanted to *double* the angular frequency ω while keeping mass the same, what would you have to do to k?

CONNECTING Ω, F, AND T

Frequency: cycles per second

$$f = \frac{\omega}{2\pi}$$

Unit: Hz

Period: seconds per cycle

$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

Unit: s

For a mass-spring oscillator, this becomes:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Don't mix them up: ω is angular frequency in rad/s, f is regular frequency in Hz. One full cycle is 2π radians, so $\omega = 2\pi f$.

FREQUENCY, PERIOD & STRATEGY

WE DO **Exercise 4: Find ω, T, and f**

A 0.50 kg mass hangs from a spring with $k = 200$ N/m. Find the angular frequency, period, and frequency.

YOU DO **Exercise 5: Working Backward**

A 2.0 kg mass on a spring oscillates with $T = 0.80$ s. Find the spring constant k.

QUICK **Exercise 6: Predict Without Calculating**

(a) If you double the mass, what happens to T? (b) If you double k? (c) If you double the amplitude?

TWO-STEP STRATEGY

At rest: $mg = kx \rightarrow k = \frac{mg}{x}$

Many real systems don't hand you k directly. First use **static equilibrium** to find k, then plug into the SHM formulas.

SKILL **Exercise 8: Spring in a Car**

A car's suspension spring compresses 0.04 m when a 60 kg person sits down.

(a) Find the spring constant k.

(b) Find the period T and frequency f of the car's bouncing.

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PUTTING IT TOGETHER

CONCEPT **Exercise 7: Scale the Mass**

Look at $T = 2\pi\sqrt{m/k}$. If you **quadruple** the mass, what exactly happens to T? Explain mathematically.

YOU DO **Exercise 9: Bungee Baby**

A baby bouncer stretches $0.12\ \mathrm{m}$ when a $7.0\ \mathrm{kg}$ baby is placed in it.
(a) Find k. **(b)** Find the period of oscillation.

(c) The baby bounces with amplitude $0.05\ \mathrm{m}$. Does this affect the period? Why or why not?

HOMEWORK

1. A spring stretches 0.20 m when a 3.0 kg mass is hung from it. (a) Find k. (b) Find T if the mass is pulled down and released.

2. A mass on a spring has $T = 1.2$ s and $k = 80$ N/m. Find the mass.

3. Two springs: Spring A has $k = 100$ N/m with a 0.5 kg mass. Spring B has $k = 400$ N/m with a 2.0 kg mass. Which has the shorter period?

Tomorrow: Pendulums! The period formula looks similar, but with one important difference—mass drops out entirely. Prediction: does a heavier pendulum swing faster, slower, or the same?