MOMENTUM DAY 2: THE IMPULSE INTEGRAL

WARMUP **Day 1 Bridge**

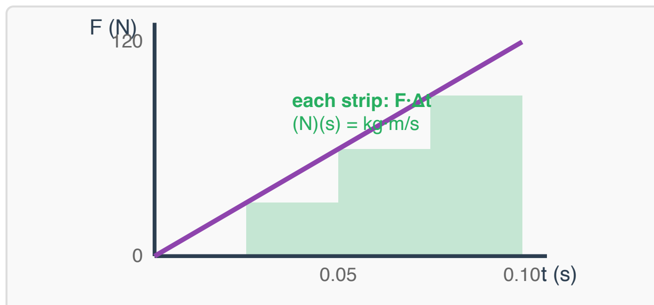
During a kick, the force on a ball ramps in a straight line from 0 up to 60 N over 0.30 s. Find the impulse delivered — area first, units of one strip attached.

Third time's the pattern. You shrank v-t strips into ∫v dt (Unit 1), then F-x strips into ∫F dx (Unit 3). Today the same limit runs on F-t: yesterday's rectangle-and-triangle areas become the impulse integral — the tool for forces that spike and fade, like every real collision.

FROM STRIPS TO THE IMPULSE INTEGRAL

Take F(t) = 1200t (N, t in seconds). Chop t = 0 → 0.10 s into strips of width Δt. Each strip adds **F·Δt** of impulse — units (N)(s) = kg·m/s. Add them up, then shrink Δt:

Strip width	Running total (left edges)
Δt = 0.05 s	(0 + 60)(0.05) = 3.0 N·s
Δt = 0.025 s	(0 + 30 + 60 + 90)(0.025) = 4.5 N·s
Δt → 0	**6.0 N·s — the exact area**



That limit IS the impulse integral: **$J = \int_0^{0.10} F(t)\,dt = 600t^2 \big|_0^{0.10} = 6.0$ N·s**.

WE DO **Exercise 1: Accumulate by Hand**

For F(t) = 1200t above: **(a)** verify the Δt = 0.025 s running total (4.5 N·s) by adding the four strips with units. **(b)** Compute the exact impulse with the antiderivative and confirm 6.0 N·s. **(c)** The strip totals undershoot no matter how skinny the strips get — what about F(t) makes left edges come up short?

NEWTON'S LAW, AS HE WROTE IT: F = DP/DT

THE DEEPER VERSION

Newton didn't write $F = ma$. He wrote that force is the rate of change of the "quantity of motion":

$$F = dp/dt$$

For constant mass, $p = mv$ differentiates to $F = m(dv/dt) = ma$ — the familiar law drops out. But the dp/dt version does more: run it backwards and $dp = F\,dt$ accumulates into **$J = \int F\,dt = \Delta p$**. The impulse-momentum theorem isn't a new law — it's Newton's second law, integrated.

Units audit: dp/dt carries (kg·m/s)/s = kg·m/s² = N. The slope of a p-t graph is a force.

Average vs instantaneous: $J/\Delta t$ gives the AVERAGE force over the whole contact. $F = dp/dt$ gives the force RIGHT NOW — the slope at one instant. A collision's peak force can be far above its average; padding works by stretching Δt to shave the peak.

WE DO **Exercise 2: Slope of the Momentum Curve**

A cart's momentum grows as $p(t) = 4t^2$ (kg·m/s when t is in seconds). **(a)** Find $F(t) = dp/dt$. **(b)** Evaluate the instantaneous force at $t = 2.0$ s. **(c)** Compute the AVERAGE force over $0 \rightarrow 2.0$ s ($\Delta p/\Delta t$) and explain why it comes out to exactly half of (b).

YOU DO **Exercise 3: Two Terms**

An object's momentum follows $p(t) = 6t^2 - 2t$ (kg·m/s when t is in seconds). **(a)** Find $F(t)$. **(b)** Evaluate the force at $t = 1.0$ s. **(c)** At what time is the force zero? (Careful: this asks about dp/dt, not p.)

REAL COLLISIONS: FORCES THAT SPIKE AND FADE

CALCULUS CORNER — THE ACCUMULATION TABLE, COMPLETE

Graph	One strip is…	Units of one strip	So the area accumulates…
v-t	$v \cdot \Delta t$	(m/s)(s) = m	**displacement**
a-t	$a \cdot \Delta t$	(m/s^2)(s) = m/s	**velocity change**
F-x	$F \cdot \Delta x$	(N)(m) = J	**work**
F-t	$\mathbf{F \cdot \Delta t}$	**(N)(s) = kg·m/s**	**impulse** (Δp)

Same force, two different strips: over distance it accumulates energy; over time it accumulates momentum. The axes decide.

WE DO **Exercise 4: A Pulse-Shaped Kick**

A kick delivers $F(t) = 3000t(0.2 - t)$ newtons (t in seconds) to a 0.40 kg ball at rest — zero at $t = 0$, peaking mid-contact, back to zero at $t = 0.20$ s. **(a)** Expand and integrate to find the total impulse over the 0.20 s contact. **(b)** Find the ball's launch speed.

CALC **Exercise 5: Peak vs Average**

For the same kick: **(a)** Find the PEAK force (it occurs at $t = 0.10$ s, the middle of the pulse). **(b)** Find the AVERAGE force, $J/\Delta t$. **(c)** Your answers should show average = ⅔ × peak. Which of the two numbers would a "the ball experienced a force of ___" news headline get wrong, and why does the difference matter for helmet design?

PAPER PRACTICE

PRACTICE PROBLEMS

P1 Straight Impulse Integral

A rocket thruster's force ramps as $F(t) = 400t$ (newtons when t is in seconds). Find the impulse delivered from $t = 0$ to $t = 0.50$ s. Write the units of one strip before you integrate.

P2 Force From Momentum

A probe's momentum follows $p(t) = 3t^3$ (kg·m/s when t is in seconds). Find the instantaneous force on the probe at $t = 2.0$ s.

P3 Piecewise Pulse

A thruster fires with a constant 50 N for the first 0.10 s, then the force ramps in a straight line down to 0 over the next 0.20 s. **(a)** Sketch the F-t graph and find the total impulse. **(b)** Find the velocity change this gives a 2.0 kg probe.

P4 Both Laws, One Object

An object's momentum follows $p(t) = 5t^2$ (kg·m/s when t is in seconds), and its mass is 2.5 kg. **(a)** Find the force at $t = 1.0$ s using $F = dp/dt$. **(b)** Find $v(t) = p/m$, differentiate for $a(t)$, and verify that ma at $t = 1.0$ s gives the SAME force. **(c)** One sentence: when would the two methods stop agreeing? (Hint: what did we hold constant?)