KINEMATICS DAY 6: INTEGRATION AS ACCUMULATION

WARMUP Day 5 Bridge

A cart starts at v_0 = 3.0 m/s. Its a-t graph ramps in a straight line from 0 up to 4.0 m/s² over 6.0 s. Find the cart's final velocity — area first, units attached, then add v_0.

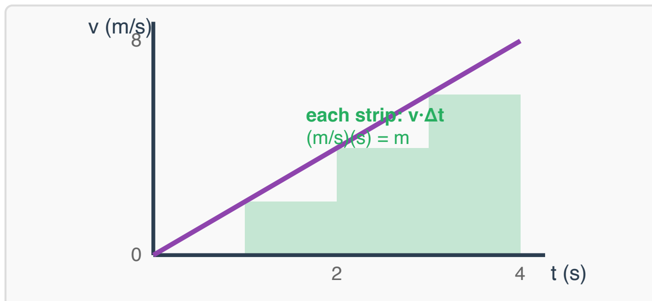
Heads-up if you're in AB Calc: your calc class won't reach integrals for months — and when it does, it may skip the best part. An integral is not "the antiderivative trick." It is a **running total**: a sum of tiny strips. You already believe this — you've been adding graph areas since Day 2. Today we just let the strips get skinny.

FROM STRIPS TO THE INTEGRAL

Take v(t) = 2t (m/s). Chop 0→4 s into strips of width Δt. Each strip adds **v·Δt** of displacement — units (m/s)(s) = m. Add them up, then shrink Δt:

Strip width	Running total (left edges)
Δt = 1 s	0 + 2 + 4 + 6 = 12 m
Δt = 0.5 s	14 m
Δt → 0	**16 m — the exact area**

That limit IS the integral: $\mathbf{\Delta x = \int_0^4 v\,dt = 16\text{ m}}$. The ∫ is a stretched S — a *Sum*.



WE DO Exercise 1: Accumulate by Hand

For v(t) = 2t above: **(a)** verify the Δt = 1 s running total (12 m) by adding the four strips with units. **(b)** Compute the exact area as a triangle and confirm 16 m. **(c)** Why does the strip total come out LOW here no matter how skinny the strips are — what about v(t) makes left edges undershoot?

COMPUTING ACCUMULATIONS: THE POWER RULE, BACKWARDS

ANTIDERIVATIVES & INITIAL CONDITIONS

For polynomials, the accumulation has a shortcut — run the power rule backwards: $\int c \cdot t^n\, dt = c \cdot t^{n+1}/(n+1)$. But an area only ever gives you the *CHANGE*. You must add where you started:

$$v(t) = v_0 + \int_0^t a\, dt \qquad x(t) = x_0 + \int_0^t v\, dt$$

Units audit every step: a-strips are (m/s²)(s) = m/s, so ∫a dt lands on velocity. v-strips are (m/s)(s) = m, so ∫v dt lands on position. The units tell you WHAT you just accumulated.

WE DO — Exercise 2: The Pattern, By Example

In words: **raise the power by one, divide by the new power.** The left pairs are worked — fill in the right pairs together, then check each answer by differentiating it back.

Given rate	Accumulated total	Given rate	Accumulated total
4	4t	5	
2t	t²	8t	
6t	3t²	9t²	
3t²	t³	4t³	

WE DO — Exercise 3: Twice Up the Ladder

A probe has acceleration **a(t) = 6t** (m/s², seconds), with v_0 = 2.0 m/s and x_0 = 0 at t = 0. **(a)** What units does the coefficient 6 carry? **(b)** Find v(t), then v at t = 2.0 s. **(c)** Find x(t), then x at t = 2.0 s.

YOU DO — Exercise 4: Don't Drop the Start

A cart's velocity is **v(t) = 3t²** (m/s, seconds), and the cart starts at x_0 = +5.0 m. **(a)** Find the displacement from t = 0 to t = 2.0 s by integrating. **(b)** Find the cart's position at t = 2.0 s. **(c)** Which of those two answers would the AREA under the v-t graph give, and why not the other?

WHAT ACCUMULATES UNDER WHICH CURVE?

CALCULUS CORNER — THE ACCUMULATION TABLE

Graph	One strip is…	Units of one strip	So the area accumulates…
v-t	$v \cdot \Delta t$	(m/s)(s) = m	**displacement** (Δx)
a-t	$a \cdot \Delta t$	(m/s²)(s) = m/s	**velocity change** (Δv)

Before you compute ANY area this year, write the units of one strip first — each strip is just a skinny rectangle under the curve. If your answer's units don't match what you claimed to accumulate, the physics is telling you to stop.

WE DO **Exercise 5: Piecewise Accumulation**

A delivery robot's v-t graph: constant +6.0 m/s for 3.0 s, then a straight-line ramp down to 0 over the next 2.0 s. **(a)** Accumulate each piece with units. **(b)** Total displacement for all 5.0 s.

YOU DO **Exercise 6: Accumulating Under a-t**

An object moves at v_0 = +5.0 m/s. Its a-t graph is constant at –2.0 m/s² for 4.0 s. **(a)** What does the area under this a-t graph accumulate — displacement or velocity change? Prove it with the units of one strip. **(b)** Find the object's final velocity. **(c)** At what time does the object momentarily stop?

CALC **Exercise 7: Name What Accumulated**

Each area below was computed correctly. Name the quantity found, with units: **(a)** area 24 under a v-t graph (axes m/s and s); **(b)** area 15 under an a-t graph (axes m/s² and s); **(c)** area 24 under a graph of *speed* vs time for a cart that turned around halfway — is 24 m its displacement or its distance?

PAPER PRACTICE

PRACTICE PROBLEMS

P1 Straight Integration

A cart's velocity is $v(t) = 3t^2$ (m/s, seconds). Find its displacement from $t = 0$ to $t = 2.0$ s. Write the units of one strip before you integrate.

P2 Accumulate, Then Add the Start

A rocket sled has $a(t) = 4t$ (m/s², seconds) and $v_0 = 3.0$ m/s. Find its velocity at $t = 3.0$ s.

P3 Two Integrations, Two Starts

A probe has $a(t) = 12t$ (m/s², seconds), $v_0 = 1.0$ m/s, $x_0 = 2.0$ m. Find **(a)** v(t) and **(b)** the probe's position at $t = 2.0$ s. Keep both initial conditions.

P4 Derive the Old Friend

Let a be constant, starting from v_0 and $x_0 = 0$. **(a)** Integrate a once (add the start) to get v(t). **(b)** Integrate v(t) to get Δx. **(c)** You just re-derived a Day 5 equation — which one? Where did the ½ come from, in strip language?