KINEMATICS DAY 5: THE KINEMATIC EQUATIONS

WARMUP **Day 4 Review**

An object moves at v_0 = +10 m/s. Its a-t graph shows a constant –2.0 m/s² for 3.0 s. Find Δv from the area — carry the units through the multiplication — and the final velocity.

CALCULUS CORNER — WHERE THE EQUATIONS COME FROM

The kinematic equations are usually handed to students as facts. Watch them **build themselves** from one assumption — constant acceleration — and the accumulation idea from Day 2:

Step 1 — accumulate the a-t graph. Constant a is a horizontal line at height a. From 0 to t it sweeps out a rectangle of area **a·t**, units (m/s²)(s) = m/s. That area is accumulated *velocity change*:

$$v(t) = v_0 + at$$

Step 2 — accumulate the v-t graph. v(t) is now a straight ramp from v_0 up to v_0 + at. Its area from 0 to t is a rectangle plus a triangle: $\mathbf{v_0 t + \tfrac{1}{2}(at)(t)}$, units (m/s)(s) = m. That area is accumulated *displacement*:

$$\Delta x = v_0 t + \tfrac{1}{2}at^2$$

Step 3 — eliminate t. Solve Step 1 for t and substitute into Step 2 (pure algebra, no accumulation):

$$v^2 = v_0^2 + 2a\Delta x$$

Every equation is an accumulation with its units built in — and each one is only as valid as its assumption: CONSTANT a.

WE DO **Exercise 1: Re-Derive With Numbers**

A cart starts at v_0 = 4.0 m/s with constant a = 3.0 m/s² for 6.0 s. **(a)** Accumulate the a-t rectangle to find Δv, then v_f. **(b)** Accumulate the v-t rectangle + triangle to find Δx. **(c)** Cross-check Δx using average velocity: $\tfrac{1}{2}(v_0 + v_f)\cdot t$. All three answers must agree — with units.

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CHOOSING THE RIGHT EQUATION

THE STRATEGY

1. List knowns and the wanted unknown (v_0, v_f, a, t, Δx). **2.** Spot the variable that appears nowhere. **3.** Use the equation missing exactly that variable.

Equation	Missing	Units check
$v_f = v_0 + at$	Δx	(m/s²)(s) = m/s ✓
$\Delta x = v_0 t + \frac{1}{2}at^2$	v_f	(m/s²)(s²) = m ✓
$v_f^2 = v_0^2 + 2a\Delta x$	t	(m/s²)(m) = m²/s² ✓

YOU DO **Exercise 2: Braking to a Stop**

A car moving at 24 m/s brakes to a stop over 48 m. **(a)** Which equation fits, and what's missing from the problem? **(b)** Find the acceleration. **(c)** Then find how long the stop takes.

YOU DO **Exercise 3: No Clock Available**

A sprinter moving at 8.0 m/s accelerates at 2.0 m/s² over a distance of 33 m. Find her final speed. *(No time given, no time wanted.)*

Two classic traps: (1) These equations require *constant acceleration* — the derivation used rectangles and straight ramps, and that's exactly where it breaks otherwise. (2) In ½at², square *only* the time: ½(2 m/s²)(3 s)² = 9 m, never ½(6)² = 18 m.

KNOWING WHEN YOU CAN'T

CALCULUS CORNER — WHEN THE KINEMATIC EQUATIONS ARE ILLEGAL

A rocket sled's velocity is **$v(t) = 12 - 3t^2$** (m/s, seconds). Differentiate: $a(t) = -6t$ — the acceleration *changes every instant*, so the kinematic equations do not apply to this motion at all. But calculus still does: a at $t = 2.0$ s is -12 m/s², and displacement is still the area under $v(t)$ — which is exactly what Day 6 teaches you to accumulate.

CALC **Exercise 4: Constant or Not?**

For each motion, differentiate to decide whether the kinematic equations are legal, and give a at $t = 2.0$ s (with sign and units): **(a)** $v(t) = 5 + 4t$ **(b)** $v(t) = 12 - 3t^2$ **(c)** $x(t) = 2t^2 + 6t$.

CHALLENGE **Exercise 5: Answer, Then Audit**

A car passes a sign moving at 12 m/s and accelerates at 2.0 m/s² for 5.0 s. **(a)** How far past the sign is it? **(b)** How fast is it going? **(c)** *Cross-check:* plug both answers into $v_f^2 = v_0^2 + 2a\Delta x$ and show the two sides agree — units and all.

PAPER PRACTICE

PRACTICE PROBLEMS

P1 From Rest, Twice Accumulated

A car starts from rest and accelerates at 3.0 m/s² for 6.0 s. Find **(a)** its final velocity (accumulate the a-t rectangle) and **(b)** how far it travels (accumulate the v-t triangle). Units through every line.

P2 The Carrier Catapult

A fighter jet leaves a carrier deck at 74 m/s after accelerating from rest over 94 m. Find the catapult's acceleration. Which equation did you choose, and what variable was missing?

P3 Average-Velocity Shortcut

A train slows steadily from 20 m/s to 5.0 m/s in 6.0 s. How far does it travel during those 6.0 s? *Hint: with constant a the average velocity is* $\frac{1}{2}(v_0 + v_f)$ *— that's the trapezoid area in disguise.*

P4 Time From a Distance

Starting from rest with a = 2.0 m/s², how long does an object take to travel 64 m?