KINEMATICS DAY 12: PROJECTILES — ANGLED LAUNCH

WARMUP **Day 11 Review**

A ball is launched horizontally at 10 m/s from a 44.1 m cliff. Find the fall time and the landing distance from the base.

RESOLVE ONCE, THEN FLY

Step 1 — resolve: $v_{0x} = v_0 \cos\theta$ (stays constant), $v_{0y} = v_0 \sin\theta$ (then free-falls).

Step 2 — fly (level ground, up positive): time to top v_{0y}/g, total flight $2v_{0y}/g$, max height $v_{0y}^2/2g$, range $v_{0x} \times$ flight time.

At the top: $v_y = 0$ but v_x keeps going — the projectile is never at rest.

WE DO **Exercise 1: The Full Flight**

A soccer ball is kicked at 20 m/s at 37° above horizontal from level ground. Find **(a)** the components, **(b)** the total flight time, **(c)** the maximum height, and **(d)** the range.

Resolve once, at the start. After Step 1, the launch speed v_0 should never appear again — only v_{0x} and v_{0y}. Mixing v_0 into a vertical equation is the #1 angled-launch error.

THE POSITION VECTOR: CALCULUS, COMPONENT BY COMPONENT

CALCULUS CORNER — DIFFERENTIATE THE WHOLE TRAJECTORY AT ONCE

Pack both axes into one **position vector**. For the 20 m/s, 37° kick ($v_{0x} = 16$, $v_{0y} = 12$):

$$\mathbf{r(t) = (16t)\,\hat{i} + (12t - 4.9t^2)\,\hat{j}} \quad \text{(meters, seconds)}$$

Differentiate component by component — that's WHY the axes don't talk to each other:

$$\mathbf{v(t) = dr/dt = 16\,\hat{i} + (12 - 9.8t)\,\hat{j}} \qquad \mathbf{a(t) = dv/dt = -9.8\,\hat{j}}$$

Everything from Days 3 and 6 works per component: derivatives give v and a; accumulations run backward. The î component never feels gravity because a has no î part.

CALC Exercise 2: Work the Vector

For r(t) = (16t) î + (12t – 4.9t²) ĵ above: **(a)** find v(t) and evaluate both components at t = 1.0 s, then compute the speed there. **(b)** At what time is $v_y = 0$, and what is the ball's speed at that instant? **(c)** Write a(t) as a vector. Which component of v does it change, and which does it leave alone?

CHALLENGE Exercise 3: No Calculator Needed

A ball launched at 26 m/s at 40° lands on level ground. **(a)** What is its landing speed? **(b)** What other launch angle at 26 m/s lands in the same spot? **(c)** Justify both in one sentence each — no computation allowed.

USE THE SYMMETRY

YOU DO Exercise 4: Complementary Angles

Two identical balls are kicked at 20 m/s — one at 30°, one at 60°. For each, find the flight time and range. What do you notice about the ranges, and which ball wins on hang time?

YOU DO Exercise 5: Steeper Kick

A ball is launched at 25 m/s at 53° above horizontal from level ground. Find the total flight time, maximum height, and range.

PAPER PRACTICE

PRACTICE PROBLEMS

P1 Vertical Component

Find the initial vertical velocity component of a launch at 30 m/s at 53° above horizontal.

P2 Hang Time

A projectile is launched with v_{0y} = 9.8 m/s on level ground. How long is it airborne?

P3 Full Analysis

A ball is launched at 19.6 m/s at 30° above horizontal on level ground. Find the flight time, maximum height, and range.

P4 · CALC Vector Workup

A projectile's position is r(t) = (8t) î + (20t – 4.9t²) ĵ (meters, seconds). **(a)** Find v(t) by differentiating each component. **(b)** At what time is the projectile at the top of its arc? **(c)** What is its SPEED at that instant — and why isn't it zero?