FORCES DAY 6: VARIABLE FORCES

WARMUP **Day 5 Review**

A 4.0 kg crate slides on a floor with $\mu_k = 0.25$ while a person pushes it forward with a constant 30 N. **(a)** Find the friction force. **(b)** Find the crate's acceleration from $\Sigma F = ma$. ($g = 9.8\ m/s^2$.)

WHEN THE FORCE WON'T HOLD STILL

Every force so far was constant, so a was constant and Unit 1's kinematic equations applied. Real forces often aren't: a rocket burns fuel, a motor ramps up, a magnet pulls harder as you approach. Newton's Second Law still holds *at every instant*:

$$a(t) = \Sigma F(t) / m$$

Divide the force function by the mass and you have the acceleration function — then Unit 1 takes over: **accumulate a(t) to get v(t), accumulate v(t) to get x(t)**, adding the starting value each time.

Units audit: N/kg = (kg·m/s²)/kg = m/s². The kinematic equations are ILLEGAL here — check by differentiating: is a constant?

WE DO **Exercise 1: The Whole Chain, Once**

A 2.0 kg cart starts from rest. A motor applies $F(t) = 12t$ (newtons, seconds). **(a)** Write a(t) — check its units. **(b)** Accumulate to find v(t), then v at t = 2.0 s. **(c)** What does the area under the a-t graph between 0 and 2.0 s represent, and what are the units of one strip?

The #1 variable-force error: plugging F(t) into $\Delta x = v_0 t + \frac{1}{2}at^2$ anyway. Those equations were DERIVED assuming constant a (Day 5 of Unit 1) — with F(t) changing, there is no single "a" to plug in. Accumulate instead.

TWO ACCUMULATIONS, TWO STARTING VALUES

WE DO **Exercise 2: Keep the Initial Velocity**

A 3.0 kg cart moving at v_0 = 4.0 m/s enters a zone where F(t) = 6t N. **(a)** Find a(t). **(b)** Find v(t) — don't drop the 4.0 m/s. **(c)** Evaluate v at t = 3.0 s.

YOU DO **Exercise 3: All the Way to Position**

For the same cart (m = 3.0 kg, v_0 = 4.0 m/s, F(t) = 6t N, x_0 = 0): **(a)** accumulate v(t) to find x(t). **(b)** Find the cart's position at t = 3.0 s. **(c)** Units check every term of x(t).

CALCULUS CORNER — READING F(T) GRAPHS

An F-t graph is an a-t graph in disguise — divide every height by m. So the area under F-t, divided by m, accumulates velocity change: each strip is (N)(s)/kg = m/s. (You'll meet the undivided area — N·s, called impulse — in Unit 4.)

GRAPHS AND A PREVIEW

YOU DO **Exercise 4: Triangle of Force**

A 5.0 kg cart starts from rest. The F-t graph ramps in a straight line from 0 up to 20 N over 4.0 s. **(a)** What is the acceleration at t = 4.0 s? **(b)** Accumulate the a-t triangle to find the cart's speed at t = 4.0 s — write the units of one strip first.

CALC **Exercise 5: When Does It Turn Around?**

A 2.0 kg probe has force F(t) = 12 – 4t (newtons, seconds) — forward at first, then reversing. **(a)** At what time is the force (and the acceleration) zero? **(b)** Is the probe's SPEED greatest before, at, or after that moment? Explain with the sign of a versus v (assume it starts from rest).

Day 15 preview: today the force depended on TIME. Some forces depend on the object's own VELOCITY — air resistance pushes back harder the faster you fall. Same law, ΣF = ma, but a whole new kind of accumulation. That's the famous one.

PAPER PRACTICE

PRACTICE PROBLEMS

P1 Force to Acceleration

A 4.0 kg cart feels $F(t) = 8t$ N. Find its acceleration at $t = 3.0$ s.

P2 Ramp Force, Final Speed

A 2.0 kg cart starts from rest under $F(t) = 4t$ N. Find its speed at $t = 5.0$ s. *(Accumulate the a-t triangle — or integrate.)*

P3 Zero-Force Moment

A 3.0 kg cart feels $F(t) = 18 - 6t$ N. **(a)** When is the acceleration zero? **(b)** What is a at $t = 1.0$ s and at $t = 5.0$ s (signs!)?

P4 The Full Chain

A 2.0 kg cart at $x_0 = 5.0$ m moving at $v_0 = 3.0$ m/s enters a zone with $F(t) = 24t$ N. Find **(a)** $v(t)$ and **(b)** the cart's position at $t = 2.0$ s. Keep both starting values.