UNIVERSAL GRAVITATION

Big Idea: The value 9.8 m/s² only applies at the surface of Earth. Newton realized that **mass attracts mass** anywhere in the universe.

PART 1: THE FORCE OF GRAVITY (F_G)

Newton's Law of Universal Gravitation

Every object in the universe attracts every other object with a force:

$$F_g = G\frac{m_1 m_2}{r^2}$$

- G: Gravitational Constant ($6.67 \times 10^{-11}\,\mathrm{N \cdot m^2/kg^2}$). A tiny number!
- r: Distance between the **centers** of the objects (not the surfaces).

WE DO Earth & Moon

Calculate F_g. Data:

- $M_E = 6.0 \times 10^{24}$ kg
- $M_m = 7.3 \times 10^{22}$ kg
- Distance $r = 3.8 \times 10^8$ m

$F_g =$ ____________ N

CONCEPT TRAP Newton's 3rd Law

Does Earth pull harder on the Moon, or does the Moon pull harder on Earth? Explain.

PART 2: THE INVERSE SQUARE LAW ($1/R^2$)

Gravity gets weaker as you get farther away. But because it is r^2, it gets weaker *fast*.

The Rules of Proportion

- If r doubles ($\times 2$), Force becomes $\frac{1}{4}$.
- If r triples ($\times 3$), Force becomes $\frac{1}{9}$.
- If r is halved ($\times \frac{1}{2}$), Force quadruples ($\times 4$).

VISUAL Sketch the Graph

Sketch F_g vs. r.

Quick Check: An astronaut weighs **800 N** on Earth's surface (distance $r = R$ from center). What is their weight at:

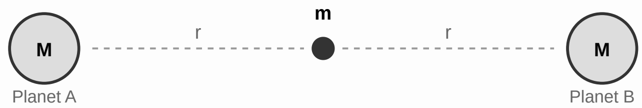
a. Distance $2R$ from center:

b. Distance $4R$ from center:

PART 3: SUPERPOSITION (NET GRAVITY)

Vector Alert: Gravity is a force vector. If multiple objects pull on you, you must **Add the Vectors** (paying attention to direction).

The Scenario: A mass m sits exactly between two identical planets.



A. The Center:

Draw the force vectors on mass m. What is the Net Force?

B. The Shift:

Planet B is moved closer (to distance $r/2$). Does the net force on m point Left or Right? Explain.

PART 4: THE GRAVITATIONAL FIELD ($\vec{g}$

The Mystery: How does Earth pull the Moon without touching it?

- **The Problem:** Newton calculated the force but never explained the mechanism. He called action-at-a-distance an "absurdity."
- **The Field Answer:** Mass creates a **Gravitational Field** in the space around it. Other masses respond to the field at their location.

g measures the field strength—the acceleration any object would experience at that location.

Calculating Field Strength

Set Weight (mg) equal to the Force of Gravity (F_g):

$$mg = G\frac{M_{source}m}{r^2}$$

Cancel the test mass (m):

$$g = \frac{GM_{source}}{r^2}$$

CONCEPT CHECK

Does g depend on the falling object?

A bowling ball and a feather are dropped on the Moon. Use the field equation to explain why they hit the ground at the same time.

Exercise: Comparing Planets

Planet X has **8× the Mass** ($8M$) and **2× the Radius** ($2R$) of Earth.
How does surface gravity on Planet X compare to Earth?

Step 1: Write the expression for g_X

Step 2: Simplify and compare to g_{Earth}

PART 5: ARTIFICIAL GRAVITY & WEIGHTLESSNESS

The *sensation* of weight comes from surfaces pushing on you, not from gravity itself.

True Gravity vs. Apparent Weight

- **True Gravity (F_g):** The pull between masses. You cannot feel this directly.
- **Apparent Weight (F_N):** The normal force from surfaces. This is what you *feel*.

In free fall, $F_N = 0$, so you feel weightless—even though gravity is still pulling.

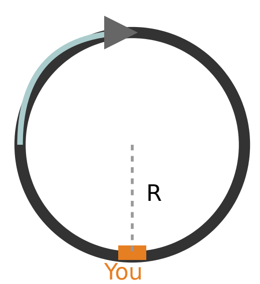
The "Fictitious" Centrifugal Force

When you spin, you feel pushed outward. Physicists call this a **Fictitious Force**—it's actually your inertia trying to go straight while the wall pushes you inward.

Einstein's insight: *If you were blindfolded, you couldn't tell the difference between this sensation and real gravity.*

Case Study: The Spinning Space Station

In sci-fi movies, space stations spin to create "Artificial Gravity." Let's analyze the physics.



1. The Free Body Diagram

You stand on the inside rim. Draw the force(s) acting on you.

Hint: What force makes you move in a circle?

2. The Connection

The Normal Force (F_N) provides the Centripetal Force. Write the equation:

3. The Goal

We want to feel Earth-like gravity. Set $F_N = mg$ and simplify:

Calculate the Speed

A space station has radius $R = 50$ m. How fast must the rim move so astronauts feel $1g$ (9.8 m/s²)?

PART 6: PRACTICE PROBLEMS

1. The Shrinking Earth

Imagine Earth kept its same mass but shrank to **half its radius** ($r \to \frac{1}{2}R$).

a. By what factor does the gravitational force on you change?

b. By what factor does g change?

2. Net Force Challenge

Consider the gravitational forces acting on star **B**.

- Calculate the ratio $F_{A\to B}/F_{C\to B}$.
- Which direction is the net force on B?

3. Einstein's Elevator

You wake up in a windowless box. You drop an apple and it falls to the floor.

Describe **two completely different** physical situations that could explain this observation.

4. The ISS Paradox (Preview for Tomorrow)

The ISS orbits 400 km above Earth's surface. (Earth's radius $R_E \approx 6.4 \times 10^6$ m)

a. Calculate g at the ISS altitude:

$r = 6.4 \times 10^6 + 0.4 \times 10^6 = 6.8 \times 10^6$ m

b. The Puzzle:

You should get $g \approx 8.7$ m/s². That's strong gravity! **So why do astronauts float?**

CALCULUS CORNER — THE SHELL THEOREM (USE IT, DON'T PROVE IT YET)

The shell theorem (Newton proved it with calculus; you get to use it): a uniform spherical shell attracts anything OUTSIDE it as if all the shell's mass sat at its center — and exerts ZERO net gravity on anything INSIDE it.

Consequences: outside a planet, g(r) = GM/r² as if the planet were a point. INSIDE a uniform planet, only the sphere beneath you counts, so g grows linearly from 0 at the center to its surface value. A tunnel through Earth would feel weaker gravity the deeper you go.