FORCES DAY 16: DRAG II — THE EXACT CURVE

WARMUP Day 15 Review

A 2.0 kg ball falls with $b = 0.98$ kg/s. **(a)** Find v_T. **(b)** The ball is currently at 10 m/s: march one step with $\Delta t = 1.0$ s to estimate its speed a second from now.

THE CURVE YOUR TABLE WAS SNEAKING UP ON

Yesterday's march traced a curve. Here is its exact formula — *presented*, the way a friend hands you an answer to check:

$$\mathbf{v(t) = v_T(1 - e^{-t/\tau})}, \quad \tau = m/b$$

τ (tau) is the motion's **time constant** — it sets how fast the fall settles toward v_T. Units check: kg/(kg/s) = s ✓.

You do NOT need to derive this (that technique arrives late in AB Calc). Your job today: verify it, then read it like a native.

THE ONE DERIVATIVE YOU NEED

Chain rule on the exponential (take this as given if your calc class hasn't reached it): $\mathbf{\frac{d}{dt} e^{kt} = k \cdot e^{kt}}$. So $\frac{d}{dt} e^{-t/\tau} = -(1/\tau)e^{-t/\tau}$.

WE DO Exercise 1: Verify It (the whole point of today)

Show the formula actually solves the physics. **(a)** Differentiate $v(t) = v_T(1 - e^{-t/\tau})$ to get dv/dt. **(b)** Separately compute $g - (b/m)\cdot v(t)$ and simplify (use $v_T = mg/b$ and $\tau = m/b$, so $v_T/\tau = g$). **(c)** Conclude: the two sides match — the curve obeys $a(v) = g - (b/m)v$ at every instant.

READING THE EXPONENTIAL LIKE A NATIVE

WHAT Τ MEANS (VALUES OF E^{-x} YOU MAY USE ALL DAY)

x	0.5	1	2	3
e^{-x}	0.607	0.368	0.135	0.050

At $t = \tau$: $v = v_T(1 - 0.368) = \mathbf{0.632 \cdot v_T}$ — one time constant reaches 63.2% of terminal. At 2τ: 86.5%. At 3τ: 95.0%. "About three time constants" ≈ "basically there."

And the tangent at $t = 0$ has slope g (check: dv/dt at $0 = v_T/\tau = g$). If the fall KEPT that first slope, it would hit v_T at exactly $t = \tau$.

WE DO **Exercise 2: The Standard Ball, Exactly**

Yesterday's ball: m = 2.0 kg, b = 0.98 kg/s (v_T = 20 m/s). **(a)** Find τ. **(b)** Find v at t = τ and at t = 2τ. **(c)** Compare v(2τ) with your hand-marched table value near t = 4 s from yesterday. Close?

YOU DO **Exercise 3: How Long to "Basically There"?**

For a 70 kg skydiver with b = 14 kg/s: **(a)** find v_T and τ. **(b)** About how long until she reaches 95% of terminal velocity? **(c)** What is her speed then?

THE GRAPH TELLS EVERYTHING

YOU DO **Exercise 4: Read a v-t Curve**

A drag-limited fall levels off at 30 m/s, and the tangent line at $t = 0$ would reach 30 m/s at $t = 3.0$ s. **(a)** Read off v_T and τ. **(b)** Find b/m for this object. **(c)** Find its speed at $t = 3.0$ s (one τ).

CALC **Exercise 5: Slope Check Without the Formula**

For any drag-limited fall from rest: **(a)** what is dv/dt at $t = 0$, in terms of g — and why must EVERY such fall start at that slope? **(b)** As $t \to \infty$, what do v and dv/dt approach? **(c)** Could the true $v(t)$ curve ever cross above v_T? Argue from $a(v)$, not the formula.

Where the formula comes from (for the curious): the technique is called separation of variables — it moves all the v's to one side of $a(v) = dv/dt$ and all the t's to the other, then accumulates both sides. Your calc class will get there; the physics didn't need to wait.

PAPER PRACTICE

PRACTICE PROBLEMS ($E^{-0.5} = 0.607$, $E^{-1} = 0.368$, $E^{-2} = 0.135$, $E^{-3} = 0.050$)

P1 Time Constant

A 3.0 kg object falls with b = 1.5 kg/s. Find τ.

P2 Speed at One τ

A falling object has $v_T = 40$ m/s. Find its speed one time constant after release from rest.

P3 The Remaining Gap

For the same object ($v_T = 40$ m/s), how far below terminal velocity is it at $t = 2\tau$?

P4 Full Workup

A 4.0 kg object falls from rest with b = 0.98 kg/s. Find **(a)** v_T, **(b)** τ, and **(c)** its speed at t = 4.08 s (hint: how many τ is that?).