FORCES DAY 15: DRAG I — TERMINAL VELOCITY

WARMUP Day 14 Review

A 0.50 kg ball on a string swings through the TOP of a vertical circle of radius 1.0 m at 4.0 m/s. Find the string tension there. (Both forces point down at the top.)

A FORCE THAT DEPENDS ON YOUR OWN SPEED

Air pushes back on anything moving through it, and pushes harder the faster you go. Our model: $\mathbf{F_{drag} = bv}$, opposing the motion, with b in kg/s (so (kg/s)(m/s) = kg·m/s² = N ✓).

For an object falling from rest, Newton's second law at every instant:

$$ma = mg - bv \quad \rightarrow \quad a(v) = g - (b/m)v$$

That completes the collection: constant forces, F(t) (Day 6), F(x) (Day 10), and now **F(v)** — the acceleration depends on how fast you're already going.

WE DO Exercise 1: The Two Ends of the Fall

A 2.0 kg ball falls from rest with b = 0.98 kg/s. **(a)** Find its acceleration at the instant of release (v = 0). **(b)** As v grows, what happens to a? **(c)** Find the speed at which a = 0 exactly.

TERMINAL VELOCITY

The speed where drag balances weight — where the fall stops speeding up:

$$v_T = mg/b \quad \text{(set } a = 0 \text{ in } a(v) = g - (b/m)v\text{)}$$

Heavier or more streamlined (small b) → faster terminal velocity. The object never exceeds v_T falling from rest — it approaches it.

STEPPING THE MOTION FORWARD — BY HAND

CALCULUS CORNER — ACCUMULATION WHEN A DEPENDS ON V

You can't integrate a(v) over time directly — you don't know v(t) yet; that's the thing you're looking for! But accumulation still works, one small strip at a time:

$$\mathbf{v_{next} = v + a(v)\cdot\Delta t} \quad \text{(use the acceleration you have NOW for one small step)}$$

Each step is one strip of the a-t graph, (m/s²)(s) = m/s — you just have to recompute the height after every step because a changed. This is exactly how computers solve motion no formula can.

WE DO **Exercise 2: March It Forward**

Same ball: m = 2.0 kg, b = 0.98 kg/s (so v_T = 20 m/s and a(v) = 9.8 – 0.49v). From rest, step with Δt = 1.0 s. The first rows are worked — fill in the last two, watching v approach (but never pass) 20 m/s.

t (s)	v (m/s)	a = 9.8 – 0.49v (m/s²)	v_{next} = v + a·Δt
0	0	9.8	9.8
1	9.8	9.8 – 4.80 = 5.00	14.8
2	14.8	9.8 – 7.25 = 2.55	17.3
3	17.3		
4			

YOU DO **Exercise 3: Read Your Table**

(a) Between which two seconds did the ball gain the MOST speed, and why? **(b)** Will the table ever reach exactly 20 m/s? Explain from a(v). **(c)** Would smaller steps (Δt = 0.1 s) make the march more or less accurate? (Unit 1's strips!)

THINKING IN A(V)

YOU DO **Exercise 4: Faster Than Terminal**

The same ball (v_T = 20 m/s) is THROWN DOWNWARD at 30 m/s. **(a)** Find a at that instant — sign and all (take down as positive). **(b)** What does the ball's speed do next? **(c)** Sketch v-t for this ball and, on the same axes, for one dropped from rest. Where do the curves end up?

CALC **Exercise 5: Going Up Is Different**

A ball moving UPWARD at speed v feels BOTH gravity and drag pulling it down (drag opposes motion). **(a)** Write |a| for the upward trip. **(b)** For m = 2.0 kg, b = 0.98 kg/s, find |a| at v = 10 m/s on the way up, and compare with |a| at 10 m/s on the way down. **(c)** With air, does a ball take longer going up or coming down? One sentence.

Tomorrow (Day 16): the exact curve your table is sneaking up on has a formula — an exponential approach to v_T. You'll verify it by differentiating, and your table will already have told you everything it means.

PAPER PRACTICE

PRACTICE PROBLEMS

P1 Terminal Velocity

A 4.0 kg drone canopy falls with $b = 1.96$ kg/s. Find its terminal velocity.

P2 Acceleration Mid-Fall

For the same object, find the acceleration at the moment its speed is 10 m/s (falling).

P3 One Step of the March

An object has $a(v) = 9.8 - 0.5v$ (m/s²; v in m/s) and is currently falling at 12 m/s. Step forward with $\Delta t = 0.5$ s: find its speed half a second from now.

P4 Working Backward From v_T

A skydiver ($m = 70$ kg) reaches a terminal velocity of 49 m/s. **(a)** Find b. **(b)** At what speed is the drag force equal to HALF her weight?