FORCES DAY 10: SPRING FORCES

WARMUP Day 9 Review

A 4.0 kg cart on a frictionless table is pulled by a string over a pulley attached to a hanging 1.0 kg mass. Treat them as one system: find **(a)** the acceleration and **(b)** the string tension. ($g = 9.8\ m/s^2$.)

HOOKE'S LAW: A FORCE THAT DEPENDS ON POSITION

Stretch or compress a spring by x from its natural length and it pushes back with

$$F_{spring} = -kx$$

k is the spring constant — how stiff the spring is — in newtons per meter. The minus sign says the force always points BACK toward natural length: stretch it, it pulls in; compress it, it pushes out. A restoring force.

Sanity check: (N/m)(m) = N. Doubling the stretch doubles the force — springs are linear (until you ruin them).

WE DO Exercise 1: Measuring k

A 0.50 kg mass hangs at rest from a vertical spring, stretching it 0.14 m. **(a)** Draw the FBD — two forces. **(b)** Use equilibrium ($\Sigma F = 0$) to find k, units included.

YOU DO Exercise 2: Force at a Stretch

A spring with k = 200 N/m is stretched 0.15 m. **(a)** Find the magnitude of the spring force. **(b)** Which way does it point?

SPRINGS MEET NEWTON'S SECOND LAW

WE DO **Exercise 3: Equilibrium Stretch**

A 2.0 kg mass hangs at rest from a spring with $k = 80$ N/m. How far is the spring stretched from its natural length?

YOU DO **Exercise 4: The Instant of Release**

A 2.0 kg cart on a frictionless track is pushed against a spring ($k = 100$ N/m), compressing it 0.30 m, and held. **(a)** Find the cart's acceleration AT THE INSTANT it is released. **(b)** Is that acceleration constant as the cart moves away? How do you know?

YOU DO **Exercise 5: Spring on an Incline**

A 3.0 kg block rests on a frictionless 30° incline, held in place by a spring ($k = 120$ N/m) parallel to the surface. Find the spring's stretch.

x is measured from NATURAL LENGTH, not from wherever the object happens to sit. For a hanging mass in equilibrium the spring is already stretched — that starting stretch matters (the last problem tonight uses it).

THE THIRD KIND OF FORCE

CALCULUS CORNER — F(X): FORCE AS A FUNCTION OF POSITION

You now have three kinds of force in your toolkit: constant (Days 1–5), time-dependent $F(t)$ (Day 6), and now **position-dependent $F(x) = -kx$**. Newton's law still holds at every instant: $a = -(k/m)x$ — the acceleration depends on WHERE the object is.

That makes the motion after release genuinely new: the further the object gets from natural length, the harder it's pulled back. Not constant a (kinematic equations illegal), and not $F(t)$ (you can't accumulate over time until you know where it is — chicken and egg!).

Unit 3 handles springs with energy (no timing needed); Unit 7 solves the motion itself — it oscillates, and it's the richest motion of the course. Today: master the FORCE.

CALC Exercise 6: Acceleration, Position by Position

A 2.0 kg cart attached to a spring ($k = 50$ N/m) is released from $x = +0.20$ m. Find the cart's acceleration **(a)** at $x = +0.20$ m, **(b)** at $x = +0.10$ m, and **(c)** at $x = 0$. **(d)** In one sentence: why can't you use $\Delta x = v_0 t + \frac{1}{2}at^2$ to time this trip?

PAPER PRACTICE

PRACTICE PROBLEMS

P1 k From Data

A 0.25 kg mass hangs at rest, stretching a spring 0.050 m. Find k.

P2 Restoring Force

A spring with k = 120 N/m is compressed 0.25 m. Find the magnitude and direction of the spring force.

P3 Two Springs, One Load

A 1.5 kg mass hangs at rest from TWO springs side by side (k_1 = 40 N/m, k_2 = 60 N/m), each stretched the same amount x. **(a)** Write the equilibrium equation — both springs pull up. **(b)** Find x.

P4 Beyond Equilibrium

A 2.0 kg mass hangs at rest from a spring with k = 98 N/m. **(a)** Find the equilibrium stretch. **(b)** The mass is pulled 0.10 m BELOW equilibrium and released. Show that the net force at that instant is k·(0.10 m) upward — the weight cancels the equilibrium part of the spring force — and find the acceleration at release.