ENERGY DAY 7: FORCE FROM POTENTIAL ENERGY

WARMUP **Day 4/6 Bridge**

A spring has $k = 200$ N/m. **(a)** Compute the stored energy U at a stretch of $x = 0.10$ m. **(b)** Sketch $U(x) = \frac{1}{2}kx^2$ from $x = -0.2$ m to $+0.2$ m. **(c)** At what x is the stored energy smallest?

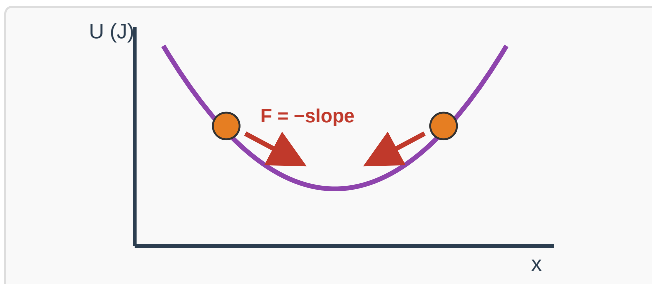
Today we invert the process. Days 3–6 you fed a force into the work integral and got potential energy out ($\frac{1}{2}kx^2$, mgh, $-GMm/r$). Today's move is the inverse: given any U(x) curve, you can read the force straight off the graph — no force diagram needed. This skill appears regularly on the Physics C exam.

FORCE POINTS DOWNHILL

From Day 3, a small bit of work changes potential energy by $dU = -F\,dx$ (energy stored is work done AGAINST the force). Solve for F:

$$F = -dU/dx$$

Read it as a landscape: the force is **minus the slope** of U(x). Uphill slope → force pushes back downhill. Steeper → stronger. Units check: J/m = N·m/m = N.



WE DO **Exercise 1: Differentiate the Landscape**

A particle has potential energy $U(x) = 2x^2 - 4x$ (joules when x is in meters). **(a)** Find $F(x) = -dU/dx$. **(b)** Evaluate the force at $x = 0$ and at $x = 2.0$ m, with signs and units. **(c)** For each, say in words which way the landscape pushes the particle.

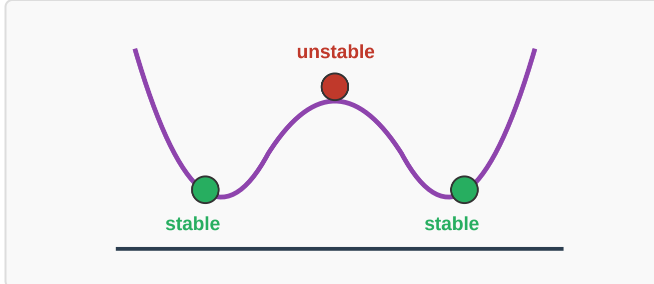
EQUILIBRIA: WHERE THE LANDSCAPE GOES FLAT

VALLEYS AND HILLTOPS

Wherever the slope is zero, the force is zero — an **equilibrium**. But not all equilibria are equal:

- **Valley bottom (U minimum):** nudge the particle either way and the force pushes it BACK. **Stable** ($d^2U/dx^2 > 0$).
- **Hilltop (U maximum):** nudge it and the force pushes it AWAY. **Unstable** ($d^2U/dx^2 < 0$).

Test: find where $dU/dx = 0$, then check the sign of the second derivative there.



WE DO **Exercise 2: Find and Classify**

A particle has $U(x) = x^3 - 3x$ (joules when x is in meters). **(a)** Find F(x). **(b)** Find both equilibrium positions. **(c)** Classify each as stable or unstable using d^2U/dx^2.

YOU DO **Exercise 3: Hooke's Law, Backwards**

Take the spring landscape $U(x) = \frac{1}{2}kx^2$. **(a)** Apply $F = -dU/dx$. What familiar law pops out? **(b)** Where is the equilibrium, and is it stable or unstable? **(c)** Explain how your answer to (b) matches what a real spring does when you displace it and let go.

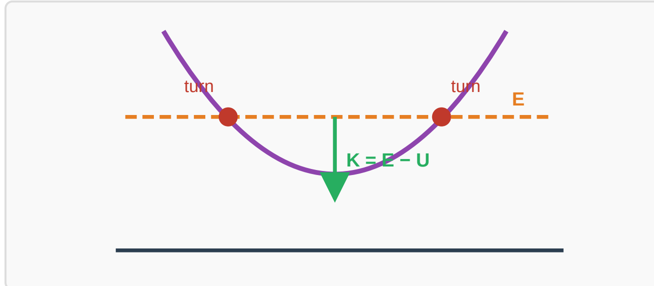
THE ENERGY LINE: TURNING POINTS AND FORBIDDEN ZONES

DRAW E AS A HORIZONTAL LINE

Total energy $E = K + U$ is conserved, so draw it as a flat line across the $U(x)$ plot. At every x:

$K = E - U(x)$ — the GAP between line and curve.

- Where the line sits above the curve: the particle can be there, moving with $K = E - U$.
- Where the line meets the curve: $K = 0$ — a **turning point**. The particle stops and reverses.
- Where the curve rises above the line: K would be negative — **forbidden**.



WE DO **Exercise 4: Turning Points by Algebra**

A 1.0 kg particle moves in $U(x) = 2x^2$ (joules when x is in meters) with total energy $E = 8.0$ J. **(a)** Find both turning points. **(b)** Find K at $x = 0$. **(c)** Find the particle's maximum speed, and state where it happens.

CALC **Exercise 5: Read the Graph Like an Examiner**

A potential energy curve falls in a straight line from $U = 10$ J at $x = 0$ to $U = 2.0$ J at $x = 4.0$ m, then stays flat. A particle starts at rest at $x = 0$ (so $E = 10$ J). **(a)** Find the force on the slope — magnitude, sign, and units. **(b)** Find the force on the flat section. **(c)** Find the particle's kinetic energy when it reaches $x = 4.0$ m.

PAPER PRACTICE

PRACTICE PROBLEMS

P1 One Valley

$U(x) = 3x^2 - 12x$ (joules when x is in meters). **(a)** Find F(x). **(b)** Find the equilibrium position and classify it.

P2 Two Equilibria

$U(x) = x^3 - 12x$ (joules when x is in meters). **(a)** Find F(x). **(b)** Find both equilibria and classify each. **(c)** Sketch the landscape well enough to show the valley and the hilltop.

P3 Turning Points

A 2.0 kg particle moves in $U(x) = 4x^2$ (joules when x is in meters) with E = 16 J. **(a)** Find the turning points. **(b)** Find the maximum speed and where it occurs.

P4 Two Old Friends From One Machine

Run $F = -dU/dx$ on the two potential energies you already own: **(a)** $U(y) = mgy$ — what force comes out, and which way does it point? **(b)** $U(r) = -GMm/r$ — differentiate with respect to r and show the machine returns the inverse-square attraction. **(c)** One sentence: why does it make sense that BOTH answers came out negative?