

ENERGY DAY 6: GRAVITATIONAL PE

Warm-Up (3 min)

A 2.0 kg book is lifted **up** 1.5 m at constant speed. ($g = 9.8 \text{ m/s}^2$.)

(a) What force does your hand exert? _____ N (c) Work done by gravity: _____ J

(b) Work done by your hand: _____ J (d) Net work on the book: _____ J

Question: You did positive work, but the book didn't speed up. Where did the energy go?

WORK DONE BY GRAVITY

THE PATTERN

Gravitational PE:

$$\Delta U_g = mg\Delta y$$

Work by gravity:

$$W_g = -\Delta U_g$$

Reference point: Choose any height as $y = 0$. Only changes in U_g matter.

The sign makes sense:

- Object moves **up** $\rightarrow \Delta U_g > 0 \rightarrow W_g < 0$
Gravity fights the motion
- Object moves **down** $\rightarrow \Delta U_g < 0 \rightarrow W_g > 0$
Gravity helps the motion

Strategy: Find ΔU_g first, then flip the sign to get W_g .

QUICK Sign Check

Motion	ΔU_g	W_g
Ball falling down	+ / - / 0	+ / - / 0
Ball thrown up (while rising)	+ / - / 0	+ / - / 0
Ball at the top (turning around)	+ / - / 0	+ / - / 0
Cart rolling on level ground	+ / - / 0	+ / - / 0

In plain English: When you lift something, you do positive work. That energy doesn't disappear —it gets *stored* as gravitational potential energy. The higher you lift, the more you store.

YOUR ENERGY INVENTORY

THREE ACCOUNTS (SO FAR)

Energy can be stored in different forms. Here are the three we've named:

Type	Symbol	Formula	"Where" It's Stored
Kinetic	K	$\frac{1}{2}mv^2$	Motion itself
Gravitational PE	U_g	mgy	Height in a gravity field
Elastic PE	U_s	$\frac{1}{2}kx^2$	Stretched or compressed spring

Energy transfers *between* these accounts. The total stays constant (for now).

CONSERVATION OF ENERGY (NO FRICTION)

THE BIG IDEA

If the only forces doing work are gravity (and springs), then:

$$K_i + U_{g,i} + U_{s,i} = K_f + U_{g,f} + U_{s,f}$$

Same total mechanical energy (for now). Cross out any accounts that are zero.

WE DO **Dropped Ball**

A ball (mass m) is dropped from rest at height h . Find its speed just before it hits the ground.

Set up the equation:

Initial state: at rest, height h

Final state: moving, height 0

Solve for v :

Result: $v =$ _____

YOU DO **Roller Coaster Drop**

A 500 kg cart starts from rest at the top of a 20 m hill. Find its speed at the bottom. (Ignore friction; $g = 9.8$ m/s².)

Energy equation:

Calculation:

Speed: $v =$ _____ m/s

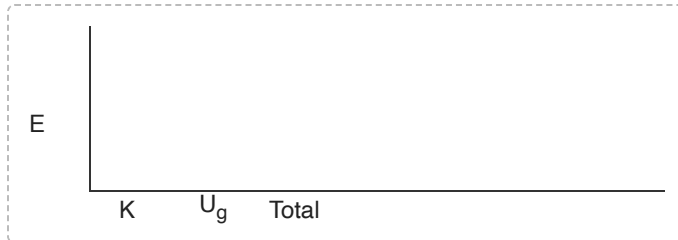
ENERGY BAR CHARTS

A visual tool: Bar charts show how energy is distributed at different moments. The total height of all bars stays constant (when no friction).

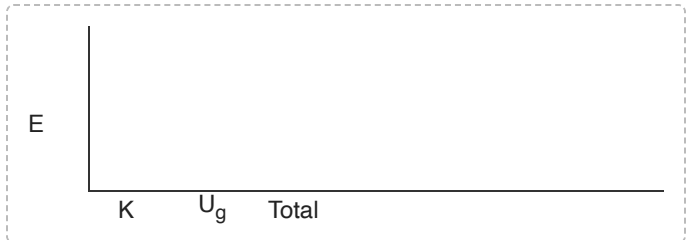
SKILL Draw the Bars

A ball is dropped from rest. Draw energy bars at each moment. Use the same total height.

Start (top, at rest)



End (bottom, moving fast)



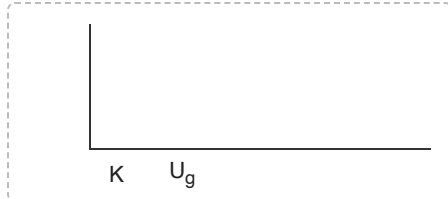
Describe in words: As the ball falls, U_g _____ while K _____.

YOU DO Ball Thrown Upward

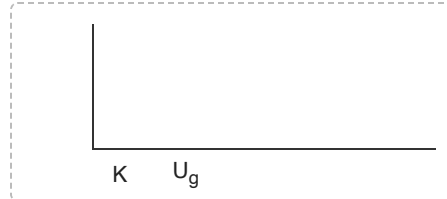
A ball is thrown straight up. It rises, slows, stops, then falls back down.

Draw bars at three moments:

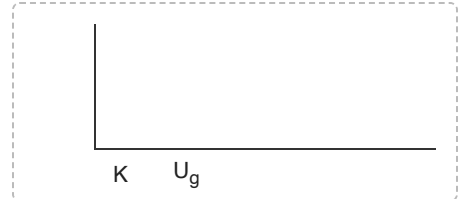
**Just after throw
(moving up fast)**



**At max height
(stopped)**



**Back at start
(moving down)**



Question: How does the speed when it returns compare to the launch speed? _____

KEY IDEA Path Doesn't Matter

Two identical balls are released from the same height. Ball A drops straight down. Ball B slides down a frictionless ramp.

Which has more speed at the bottom?

Explain using energy:

EXIT TICKET & HOMEWORK

EXIT Quick Check

A 2.0 kg object is lifted 3.0 m straight up at constant speed. ($g = 9.8 \text{ m/s}^2$.)

(a) $\Delta U_g =$ _____ J

(c) Work done by hand = _____ J

(b) $W_g =$ _____ J

(d) $\Delta K =$ _____ J

HOMEWORK

1 Conceptual

A pendulum swings from point A (high) to point B (low) to point C (high again). No friction.

(a) At which point is K maximum? _____

(c) At which point is speed maximum? _____

(b) At which point is U_g maximum? _____

(d) How do heights at A and C compare? _____

3 Working Backward

A ball is thrown straight up with speed 12 m/s. How high does it rise? ($g = 9.8 \text{ m/s}^2$.)

Hint: At max height, $K = 0$. Set initial $y = 0$.

Answer: $h =$ _____ m

CALCULUS CORNER — WHEN MGH ISN'T ENOUGH: $U = -GMm/R$

$U_g = mgy$ treats gravity as a CONSTANT force — honest near Earth's surface, where g barely changes.

Leave the surface and gravity weakens: $F = GMm/r^2$. What is the potential energy then? Same playbook as Day 3 — accumulate the work integral. Set $U(\infty) = 0$ and let gravity pull a mass in from infinity to a distance r :

$$W_{grav} = \int_{\infty}^r \left(-\frac{GMm}{r'^2} \right) dr' = \left[\frac{GMm}{r'} \right]_{\infty}^r = \frac{GMm}{r} \quad \Rightarrow \quad U(r) = -W_{grav} = -\frac{GMm}{r}$$

(The minus sign inside the integral says the force points toward smaller r ; the work comes out positive, as it should for a helpful pull.) Sanity checks: U is negative everywhere and climbs toward zero as $r \rightarrow \infty$ — every orbiting object sits in an energy well. And near the surface, the well's local slope reproduces the familiar formula: $\Delta U \approx \frac{GMm}{R^2} \Delta y = mg \Delta y$.