ENERGY DAY 5: KINETIC ENERGY

Part 2 of 2 — Testing whether stored work becomes motion

□ **Warm-Up: Recall Yesterday** — Pull out your Day 4 lab packet. Write your values:

k = ________________ N/m x = ________________ m W = ________________ J v ≈ ____________ m/s

Today's question: Does stored work become motion? We need a formula for "motion energy."

THE "MOTION ENERGY" FORMULA

KINETIC ENERGY (K)

$$K = \frac{1}{2}mv^2$$

m = mass (kg), **v** = speed (m/s), **K** = Kinetic Energy (J). Scalar, always ≥ 0.

DERIVATION **Build $W_{net} = \Delta K$ from $F = ma$ and kinematics**

Constant net force F_{net} acts on mass m over distance d, changing speed from v_i to v_f.

1) Work: $W_{net} = F_{net} \cdot d$. **2)** Sub $F = ma$: $W_{net} = (\_\_\_\_\_\_) \cdot d$

3) From $v_f^2 = v_i^2 + 2ad$, isolate: $ad =$ ____________

4) Combine: $W_{net} = m(ad) = m\,(\_\_\_\_\_\_\_\_\_\_\_\_)$

5) Distribute: $W_{net} = \_\_\_\_ mv_f^2 \; - \; \_\_\_\_ mv_i^2$

Definition: $K \equiv \frac{1}{2}mv^2$. Then: $W_{net} = \Delta K = K_f - K_i$.

Checkpoint: If W_{net} is negative, what happens to K (and speed)? ______________________________

THE WORK-ENERGY THEOREM

$$W_{net} = \Delta K = K_f - K_i$$ — Net work = change in kinetic energy.

TESTING YESTERDAY'S PREDICTION

TESTING THE CLAIM

Yesterday you found the spring stores $W = \frac{1}{2}kx^2$ J. **Claim:** if that stored work all becomes KE:

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2 \quad \rightarrow \quad v = \sqrt{\frac{k}{m}}\,x$$

PREDICT — Calculate Expected Speed

Using your k and the cart mass, predict the launch speed.

k = ______________ N/m   x = ______________ m

m = ______________ kg

Calculate $v = \sqrt{\frac{k}{m}}\,x$:

Predicted v: ______________ m/s

DATA — Measure Actual Speed

Trial	Stretch x (m)	Measured v (m/s)	Predicted v (m/s)
1			
2 (double x)			

Percent difference: ________________________________

ANALYZE — The v² Test

Doubling stretch → **4x** stored energy ($W \propto x^2$). If all becomes KE ($K \propto v^2$), speed should increase by factor ______.

From data: $v_2/v_1 =$ __________   Close to 2?   **If ratio < 2, where did the "missing" energy go?** ______

Verdict: Does stored spring work become KE? ______.   Predicted vs. actual agree to within ______%.

PRACTICE: USING THE WORK-ENERGY THEOREM

We now have a powerful tool: **net work causes changes in kinetic energy.**

WE DO **Stopping Distance**

A car traveling at speed v skids to a stop in distance d.
If the car were traveling at **2v**, how far would it slide before stopping? (Assume same friction force.)

Reasoning steps:

1. Doubling speed $\to$ _______ times the kinetic energy.
2. Friction removes energy by work: $W = Fd$.
3. To remove $4\times$ energy with the same force, you need _______ times the distance.

Answer:

__________ **d**

YOU DO **Calculations**

1. A 1000 kg car accelerates from 0 to 20 m/s. Calculate the net work done on the car.

2. The same car accelerates from 20 m/s to 40 m/s. Calculate the net work done.

3. Why does it take so much more work to gain the second 20 m/s?

Common error: ΔK depends on $v_f^2 - v_i^2$, not $(v_f - v_i)^2$.
Going from 0→20 changes v^2 by 400. Going from 20→40 changes v^2 by 1200!

CALCULUS CORNER — THE THEOREM WITHOUT "CONSTANT FORCE" (READ-ONLY)

The page-1 derivation leaned on $v_f^2 = v_i^2 + 2ad$, which only holds for constant acceleration. The calculus version drops that crutch. Start from the work integral and substitute twice:

$$W_{net} = \int F_{net}\,dx = \int ma\,dx = \int m\frac{dv}{dt}\,dx = \int m\frac{dx}{dt}\,dv = \int_{v_i}^{v_f} mv\,dv = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

Two substitutions deep, the work-energy theorem holds for ANY force profile — constant, spring, drag, anything. Read the chain today; you'll produce it yourself later. Notice where $\frac{1}{2}mv^2$ is born: it is the accumulation of $mv\,dv$.

HOMEWORK

1 Arrow Comparison

Two identical arrows: A at speed v, B at speed $3v$. (a) Which has more KE? (b) How many times more?

2 Sign of Work

A ball is thrown straight up. While rising:

(a) Is gravity doing positive or negative work? __________ (b) Is KE increasing or decreasing? __________

(c) Do your answers match $W_{net} = \Delta K$? Explain.

3 Bullet vs. Gun

A gun fires a bullet. Force on bullet = force on gun (Newton's 3rd), but the bullet travels far down the barrel while the gun barely recoils. Which gains more KE? Use $W = Fd$ to explain.

4 Hammer Time

You swing a hammer (mass m) at a nail at speed v. It drives the nail distance d into wood. Write an expression for the **average force** F on the nail. (Hint: nail does negative work on hammer.)