

ENERGY DAY 11: POWER

Warm-Up (3 min): Same Stairs, Different Time

Two students, **Alice** and **Bob**, have the exact same mass (60 kg).

- Alice walks up a flight of stairs with vertical rise $h = 3.0$ m in **10 s**.
- Bob runs up the same stairs in **2 s**.

1. Who gains more gravitational potential energy, ΔU_g ?

2. What quantity is *missing* from your energy analysis that distinguishes Alice and Bob?

Hint: The missing idea is “how fast energy changes.” Today we give that idea a name and a formula.

DEFINING POWER

RATE OF ENERGY TRANSFER

Power (P) tells how quickly energy is transferred or work is done.

$$P = \frac{W}{t} = \frac{\Delta E}{t}$$

Units:

- SI Unit: **Watt (W)**
- $1 \text{ W} = 1 \text{ J/s}$

Conversions:

- 1 horsepower (hp) $\approx 746 \text{ W}$
- $1 \text{ kW} = 1000 \text{ W}$

WE DO The Lightbulb

A "60 Watt" lightbulb uses electrical energy to create light (and heat).

How much energy (in Joules) does it use in 1 minute?

Knowns:

$$P = \underline{\hspace{2cm}}$$

$$t = \underline{\hspace{2cm}}$$

Calculation:

Unit Trap: The Kilowatt-hour (kWh)

Your electric bill measures "kWh". Is this **Power** or **Energy**?

$$(\text{Power}) \times (\text{Time}) = (\text{Energy}) \quad \Rightarrow \quad (\text{kW}) \times (\text{hr}) = \text{kWh}$$

So **kWh is a unit of energy** (not power).

POWER, FORCE, AND VELOCITY

DERIVATION

Start with the definition:

$$P = \frac{W}{t}$$

Substitute $W = Fd$:

$$P = \frac{Fd}{t}$$

Regroup:

$$P = F \left(\frac{d}{t} \right)$$

But $\frac{d}{t} = v$, so:

$$P = Fv$$

(More generally: $P = \vec{F} \cdot \vec{v} = Fv \cos \theta$.)

CALCULUS CORNER — INSTANTANEOUS POWER

$P = W/t$ is an AVERAGE rate. When the rate itself changes — a car pulling away, a rocket throttling up — shrink the time window: $P = \frac{dE}{dt}$, the slope of the energy-versus-time curve. The same shrink turns the derivation at left into $P = \frac{dW}{dt} = F \frac{dx}{dt} = Fv$, where v is now the instantaneous speed.

And the accumulation view: watts accumulate over time into joules, $E = \int P dt$ — each strip under a P-t curve is (J/s)(s) = J. That is exactly why the kilowatt-hour on page 1 is an energy unit: it is a power times a time.

WE DO Car on a Highway

A car is traveling at a constant speed of 30 m/s. Air resistance exerts a force of 600 N against the car.

How much power must the engine deliver to maintain this speed?

Net force idea: If speed is constant, then $a = 0$ and **Power:** $P = Fv$

$F_{\text{engine}} = F_{\text{air}}$.

Answer: _____ W (_____ hp)

YOU DO The Elevator

A 1000 kg elevator is lifted upward at a constant speed of 4.0 m/s. Calculate the power output of the lift motor. ($g = 9.8 \text{ m/s}^2$.)

Force required: (Draw FBD: tension up, gravity down)

$F_T =$ _____ N

Power: $P = Fv$

$P =$ _____ W

SPORTS, CALORIES, AND POWER

MECHANICAL VS FOOD ENERGY

Mechanical energy is the work you do on your body (or on an object).

Food energy is chemical energy your body uses to make that work (and lots of heat).

Food “Calories”

$$1 \text{ Calorie (food)} = 1 \text{ kcal} = 4184 \text{ J}$$

Efficiency (typical)

$$\eta \approx 0.25$$

$$E_{\text{food}} \approx \frac{W_{\text{mech}}}{\eta}$$

A cycling “power meter” reports **mechanical power** (Watts). Your body’s food power is bigger because you are not perfectly efficient.

WE DO Bike Power Meter → Calories

A cyclist holds a steady **mechanical** power of $P_{\text{mech}} = 250 \text{ W}$ for **30 minutes**.

- How much mechanical energy (work) is done?
- Estimate food Calories burned if $\eta = 0.25$.

(a) Mechanical energy: $E_{\text{mech}} = Pt$

(b) Food Calories: $E_{\text{food}} = E_{\text{mech}}/\eta$, then divide by 4184

YOU DO Stairs Sprint

A 60 kg runner climbs stairs with vertical rise $h = 3.0 \text{ m}$ in $t = 2.0 \text{ s}$. ($g = 9.8 \text{ m/s}^2$.)

- Find the mechanical work gained (mgh).
- Find the average mechanical power.
- Estimate food Calories used if $\eta = 0.25$.

HOMEWORK

1 Work vs Power

Motor A lifts a 10 kg box 5 m in 2 s. Motor B lifts a 20 kg box 5 m in 4 s. ($g = 9.8 \text{ m/s}^2$.)

(a) Which motor did more **work**? ☐ A ☐ B ☐ Same (b) Which used more **power**? ☐ A ☐ B ☐ Same

Explain your reasoning for power:

2 Hill Climb

A cyclist (mass 70 kg) rides up a hill at a constant speed of 5.0 m/s. The hill is 200 m long and 15 m high. Friction/air resistance is negligible. ($g = 9.8 \text{ m/s}^2$.)

- What is the change in gravitational PE?
- How much time does it take to climb the hill?
- Calculate the cyclist's average power output.

3 Force and Speed

A jet engine produces 50,000 N of thrust. How much power does it deliver when the plane is flying at 250 m/s?

Answer in MegaWatts (MW): _____ MW

4 Efficiency (Bonus)

An electric motor uses 500 W of electrical power but only produces 400 W of mechanical lifting power. What happened to the other 100 W?