POTENTIAL DAY 5: POTENTIAL BY INTEGRATION

WARMUP Day 4 of Unit 8, Remembered

The FIELD of a ring on its axis needed a symmetry argument and a $\cos\theta$ projection. The POTENTIAL is a scalar. Predict: which integral will be easier, and why?

SCALAR DQ INTEGRALS

$V = \int k \cdot dq/r$ — no components, no angles, ever.

Ring, on axis: every dq the same distance out → $V = kQ/\sqrt{z^2 + R^2}$. One division. **At the center:** $V = kQ/R$ while $E = 0$ — cancelled vectors, accumulated scalars.

Disk, on axis: integrate rings → $V = 2\pi k\sigma(\sqrt{z^2 + R^2} - z)$.

And E when you need it: differentiate! $E_z = -dV/dz$ reproduces Unit 8's field results — often the fastest route.

WE DO Exercise 1: Ring Numbers

Ring: $R = 1.5$ m, $Q = +5.0\ \mu C$. **(a)** V on the axis at $z = 2.0$ m (3-4-5 hypotenuse!). **(b)** V at the center. **(c)** E at the center — and one sentence on how (b) and (c) can coexist.

DISKS, AND THE DIFFERENTIATION SHORTCUT

WE DO **Exercise 2: Disk Numbers**

Disk: $R = 3.0$ m, $\sigma = 2.0\ \mu C/m^2$. Find V on the axis at $z = 4.0$ m: $V = 2\pi k\sigma(\sqrt{z^2+R^2} - z) = 2\pi k\sigma($_________ – _________$)$ = _________ V.

YOU DO **Exercise 3: Non-Uniform Rod (Scalar Style)**

A rod on [0, 2.0 m] has $\lambda(x) = 3.0x\ \mu C/m^2$. Point P sits on the axis at $x = -1.0$ m, one meter left of the rod's near end — so a slice dq at position x is a distance $(1 + x)$ from P. Set up $V = \int k(3x)dx/(1+x)$ — write the integral with limits; evaluate ONLY the total charge $Q = \int \lambda dx$ as a check of your dq.

CALC **Exercise 4: Differentiate Back to E**

Take the ring's $V(z) = kQ(z^2 + R^2)^{-1/2}$ and compute $E_z = -dV/dz$. Show the chain rule delivers $kQz/(z^2 + R^2)^{3/2}$ — exactly Unit 8's hard-won field result, in three lines.

PAPER PRACTICE

PRACTICE PROBLEMS

P1 Ring V

Ring $R = 2.0$ m, $Q = +8.0\ \mu C$: V on the axis at $z = 1.5$ m.

P2 Center vs Axis

For P1's ring, compute V at the center and the RATIO V(center)/V(z = 1.5 m).

P3 Disk V

Disk $R = 4.0$ m, $\sigma = 1.0\ \mu C/m^2$: V on the axis at $z = 3.0$ m.

P4 V → E, Again

Differentiate the disk's $V(z) = 2\pi k\sigma(\sqrt{z^2+R^2} - z)$ with respect to z and show $E_z = 2\pi k\sigma(1 - z/\sqrt{z^2+R^2})$ — Unit 8's disk field. Which derivation was less work, the Unit-8 ring-stacking or this one?