

MAGNETISM DAY 5: AMPÈRE'S LAW

WARMUP Gauss, But for Loops

Yesterday's straight wire: $B = \mu_0 I / 2\pi d$. Multiply that field by the circumference of the circle it lives on: $B \cdot (2\pi d) = \underline{\hspace{2cm}}$. Everything cancels except $\underline{\hspace{2cm}}$. That result — the product never depends on d — is today's law.

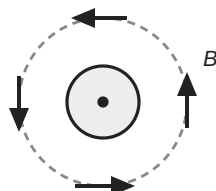
AMPÈRE'S LAW

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}}$$

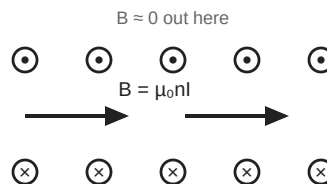
Wire (outside, $r > a$): $B \cdot 2\pi r = \mu_0 I \rightarrow B = \mu_0 I / 2\pi r$. **Inside a uniform wire ($r < a$):** only the fraction r^2/a^2 is enclosed $\rightarrow B = \mu_0 I r / 2\pi a^2$ — grows linearly, peaks at the surface.

Solenoid: a rectangular loop straddling the side gives $\mathbf{B} = \mu_0 n \mathbf{I}$ inside (n = turns per meter), ≈ 0 outside — magnetism's uniform-field device.

Like Gauss: always true, only USEFUL when symmetry makes B constant on the path.



wire, I out of the page



solenoid, cut lengthwise ($\odot$ toward you, $\otimes$ away)

The two symmetry workhorses. Exercise 4 asks YOU to add the rectangular loop to the solenoid picture.

WE DO Exercise 1: Outside the Wire

A wire carries 10 A. **(a)** Write Ampère for a circle at $r = 2.0$ m and solve: $B = \underline{\hspace{2cm}}$ μT . **(b)** At $r = 4.0$ m: $B = \underline{\hspace{2cm}}$ μT (halved — $1/r$, not $1/r^2$).

WE DO Exercise 2: Inside the Wire

A thick wire (radius $a = 2.0$ mm) carries 10 A spread uniformly. At $r = 1.0$ mm: **(a)** enclosed current = $I \cdot (r^2/a^2)$ = _____. **(b)** $B = \mu_0 I_{\text{enc}} / 2\pi r =$ _____ μT . **(c)** Where is B largest — center, surface, or far away?

YOU DO Exercise 3: Solenoid Numbers

A solenoid has 1000 turns per meter carrying 2.0 A. Find B inside (in mT), and state what happens to it just outside the coil.

CALC Exercise 4: Derive the Solenoid Field

Draw a rectangular Amperian loop of width w with one side inside the solenoid (parallel to B) and one side outside ($B \approx 0$). Show that $\oint \mathbf{B} \cdot d\mathbf{l} = Bw$, that $I_{\text{enc}} = nwl$, and conclude $B = \mu_0 nI$. Which two sides contribute nothing, and why?

PAPER PRACTICE

PRACTICE PROBLEMS

P1 Outside

B at $r = 0.10$ m from a wire carrying 25 A, in μT ?

P2 Solenoid

A solenoid with $n = 2000$ turns/m carries 1.5 A. Find B inside, in mT.

P3 Design

You need $B = 5.0$ mT inside a solenoid running at 2.0 A. How many turns per meter?

P4 The Full Profile

For a thick wire of radius a carrying I : write $B(r)$ for $r < a$ and $r > a$, verify both give the same value at $r = a$, and sketch B versus r from 0 to $4a$.