

MAGNETISM DAY 4: BIOT–SAVART

WARMUP Where Does B Come From?

Yesterday two parallel wires attracted each other — wire 1's current put a field at wire 2's location. The demo that discovered this: a compass beside a wire snaps sideways when the current switches on (Oersted, mid-lecture, 1820); carried around the wire, the needle traces CIRCLES. Sketch those circular field lines, right-hand grip rule (thumb along I, fingers curl along B) setting their direction.

BIOT–SAVART: THE DQ INTEGRAL OF MAGNETISM

$$d\mathbf{B} = (\mu_0/4\pi) I d\mathbf{l} \times \hat{\mathbf{r}} / r^2 \quad (\mu_0 = 4\pi \times 10^{-7} \text{ T}\cdot\text{m/A, so } \mu_0/4\pi = 10^{-7})$$

Loop center: every $d\mathbf{l} \perp \hat{\mathbf{r}}$ at constant $r = R \rightarrow \mathbf{B} = \mu_0 I/2R$ (N turns: multiply by N; an arc: multiply by its fraction of the circle).

Infinite straight wire (the other landmark): $\mathbf{B} = \mu_0 I/2\pi d$ — circles around the wire, right-hand grip rule.

WE DO Exercise 1: Loop Numbers

A single loop, $R = 0.5 \text{ m}$, $I = 5.0 \text{ A}$. **(a)** B at the center $= \mu_0 I/2R = \text{_____} \mu\text{T}$. **(b)** Wound as a 100-turn coil instead: $B = \text{_____} \mu\text{T}$.

WE DO Exercise 2: The Straight Wire

A power line carries 10 A. **(a)** B at $d = 0.05$ m: $\mu_0 I / 2\pi d = (2 \times 10^{-7})(I)/d = \text{_____} \mu\text{T}$. **(b)** Compare to Earth's $\sim 50 \mu\text{T}$ field — could a compass detect this wire?

YOU DO Exercise 3: The Half Loop

A wire is bent into a SEMICIRCLE of radius 0.2 m carrying 6.0 A. Find B at the center. (What fraction of the full loop's field survives, and why does the symmetry argument still hold?)

CALC Exercise 4: Set Up the Center Integral

For the full loop, write $B = \int (\mu_0 / 4\pi)(I \, dl \sin 90^\circ) / R^2$ around the circumference, pull the constants out, and evaluate $\int dl = 2\pi R$ to reach $\mu_0 I / 2R$. Note where each simplification uses the geometry.

PAPER PRACTICE

PRACTICE PROBLEMS

P1 Loop Center

$R = 0.25 \text{ m}$, $I = 4.0 \text{ A}$, single loop: find B at the center in μT .

P2 Near a Wire

Find B at 0.02 m from a wire carrying 15 A , in μT .

P3 Coil Design

How many turns (radius 0.10 m , $I = 2.0 \text{ A}$) are needed for $B = 1.26 \text{ mT}$ at the center?

P4 Quarter Arc

A quarter-circle arc ($R = 0.1 \text{ m}$) carries 8.0 A . Find B at the center of curvature — and state why the two straight feed-in wires pointing RADIALLY at the center contribute nothing (look at $d\mathbf{l} \times \hat{\mathbf{r}}$).